XI. Resonant absorption and a.c. Stark shift

Equations to be used:

$$|i\rangle = |n_i\, L_i\, S\, J_i\, I\, F_i M_{Fi}\rangle, \qquad |f\rangle = |n_f\, L_f\, S\, J_f\, I\, F_f M_{Ff}\rangle, \qquad M_{Ff} = M_{Fi} + \mu$$

$$d_{fi} = (-1)^{L_f+S+J_i+J_f+I+F_i} \sqrt{(2J_f+1)(2J_i+1)(2F_i+1)}\, C^{F_f M_{Ff}}_{F_i M_{Fi} 1\mu} \times$$

$$\times \begin{Bmatrix} J_i & I & F_i \\ F_f & 1 & J_f \end{Bmatrix} \begin{Bmatrix} L_i & S & J_i \\ J_f & 1 & L_f \end{Bmatrix} \langle n_f L_f || \hat{d} || n_i L_i \rangle$$

$$\langle n_f L_f S J_f I F_f || \hat{d} || n_i L_i S J_i I F_i \rangle = (-1)^{J_f+I+F_i+1} \sqrt{(2F_f+1)(2F_i+1)} \times$$

$$\times \begin{Bmatrix} J_i & I & F_i \\ F_f & 1 & J_f \end{Bmatrix} \langle n_f L_f S J_f || \hat{d} || n_i L_i S J_i \rangle$$

$$\langle n_f L_f S J_f || \hat{d} || n_i L_i S J_i \rangle = (-1)^{L_f+S+J_i+1} \sqrt{(2J_f+1)(2J_i+1)} \begin{Bmatrix} L_i & S & J_i \\ J_f & 1 & L_f \end{Bmatrix} \langle n_f L_f || \hat{d} || n_i L_i \rangle$$

$$C^{c\gamma}_{a\alpha\,1\beta}$$

c	$\beta = 1$	$\beta = 0$	$\beta = -1$
$a+1$	$\left[\frac{(c+\gamma-1)(c+\gamma)}{(2c-1)\,2c}\right]^{1/2}$	$\left[\frac{(c+\gamma)(c-\gamma)}{(2c-1)\,c}\right]^{1/2}$	$\left[\frac{(c-\gamma-1)(c-\gamma)}{(2c-1)\,2c}\right]^{1/2}$
a	$-\left[\frac{(c+\gamma)(c-\gamma+1)}{2c\,(c+1)}\right]^{1/2}$	$\frac{\gamma}{[c\,(c+1)]^{1/2}}$	$\left[\frac{(c+\gamma+1)(c-\gamma)}{2c\,(c+1)}\right]^{1/2}$
$a-1$	$\left[\frac{(c-\gamma+1)(c-\gamma+2)}{(2c+2)(2c+3)}\right]^{1/2}$	$-\left[\frac{(c+\gamma+1)(c-\gamma+1)}{(c+1)(2c+3)}\right]^{1/2}$	$\left[\frac{(c+\gamma+2)(c+\gamma+1)}{(2c+2)(2c+3)}\right]^{1/2}$

$$C^{c\gamma}_{a\alpha\,b\beta} = (-1)^{a+b-c}\, C^{c\gamma}_{b\beta\,a\alpha} = (-1)^{a-\alpha}\sqrt{\frac{2c+1}{2b+1}}\, C^{b-\beta}_{a\alpha\,c-\gamma} = (-1)^{a-\alpha}\sqrt{\frac{2c+1}{2b+1}}\, C^{b\beta}_{c\gamma\,a-\alpha} =$$

$$= (-1)^{b+\beta}\sqrt{\frac{2c+1}{2a+1}}\, C^{a-\alpha}_{c-\gamma\,b\beta} = (-1)^{b+\beta}\sqrt{\frac{2c+1}{2a+1}}\, C^{a\alpha}_{b-\beta\,c\gamma}.$$

$$C^{c\gamma}_{a\alpha\,b\beta} = (-1)^{a+b-c}\, C^{c-\gamma}_{a-\alpha\,b-\beta}.$$

$$\sum_{\alpha\beta\delta} C^{c\gamma}_{a\alpha\, b\beta} C^{e\varepsilon}_{d\delta\, b\beta} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_1 \Pi_{cd} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\alpha\beta\delta} C^{a\alpha}_{b\beta\, c\gamma} C^{d\delta}_{b\beta\, e\varepsilon} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_1 \frac{\Pi_{add}}{\Pi_e} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\sigma\beta\delta} C^{c\gamma}_{b\beta\, a\alpha} C^{e\varepsilon}_{b\beta\, d\delta} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_2 \Pi_{cd} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\alpha\beta\delta} (-1)^{a-\alpha} C^{c\gamma}_{a\alpha\, b\beta} C^{e\varepsilon}_{d\delta\, b\beta} C^{f\varphi}_{d\delta\, a-\alpha} = \varkappa_1 \Pi_{cf} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\alpha\beta\delta} (-1)^{b+\beta} C^{c\gamma}_{a\alpha\, b\beta} C^{d\delta}_{b-\beta\, e\varepsilon} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_1 \frac{\Pi_{cdd}}{\Pi_e} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\alpha\beta\delta} (-1)^{a-\alpha} C^{c\gamma}_{b\beta\, a\alpha} C^{e\varepsilon}_{b\beta\, d\delta} C^{f\varphi}_{d\delta\, a-\alpha} = \varkappa_2 \Pi_{cf} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

$$\sum_{\alpha\beta\delta} (-1)^{b+\beta} C^{a\alpha}_{b-\beta\, c\gamma} C^{e\varepsilon}_{d\delta\, b\beta} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_1 \Pi_{ad} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b & c \\ e & f & d \end{Bmatrix},$$

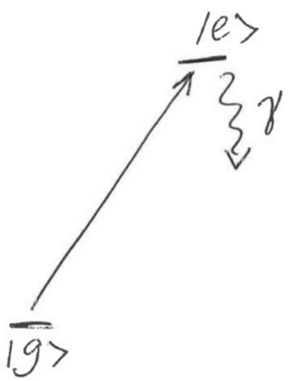
$$\sum_{\alpha\beta\delta} (-1)^{a-\alpha} C^{b\beta}_{a\alpha\, c-\gamma} C^{e\varepsilon}_{d\delta\, b-\beta} C^{d\delta}_{a\alpha\, f\varphi} = \varkappa_1 \Pi_{bd} C^{e\varepsilon}_{c\gamma\, f\varphi} \begin{Bmatrix} a & b. & c \\ e & f & d \end{Bmatrix},$$

$$\varkappa_1 = (-1)^{b+c+d+f},$$

$$\varkappa_2 = (-1)^{a+b+e+f}$$

These expressions are easily derived from the sum of products of four CG symbols, that follows from the definition of the 6*j*-symbol. Multiply both sides by $C^{e\varepsilon}_{c\gamma\, f\varphi}$ and take a sum over γ and φ.

The resonant excitation



Schrödinger equation describing excitation from the ground state $|g\rangle$ to the optically excited state $|e\rangle$.

$$|\Psi(t)\rangle = a_g(t)|g\rangle + a_e(t)|e\rangle$$

$$i\dot{a}_g = -\frac{(\vec{d}_{eg}\vec{E})^*}{\hbar} a_e$$

$$i\dot{a}_e = -(\Delta+i\gamma)a_e - \frac{\vec{d}_{eg}\vec{E}}{\hbar} a_g$$

Detuning $\Delta = \omega - \omega_{eg}$

2γ is the decay rate (into non-observable states, formally speaking).

Without defining other quantum numbers, $|e\rangle = |F_e M_e\rangle$, $|g\rangle = |F_g, M_g\rangle$

$$M_e = M_g + \mu, \quad \mu = 0, \pm 1$$

$$d_{eg} = \frac{C^{F_e M_e}_{F_g M_g 1\mu}}{\sqrt{2F_e+1}} \langle F_e \| d \| F_g \rangle$$

Assume that $\vec{E}$ contains only the polarization component that drives the transition $|g\rangle \leftrightarrow |e\rangle$:

$$\vec{E} = \vec{e}^{\,-\mu} E_{-\mu} e^{-i\omega t} + c.c. =$$
$$= \vec{e}_\mu (-1)^\mu E_{-\mu} e^{-i\omega t} + c.c.$$

Intensity $I = 2c\varepsilon_0 |E_{-\mu}|^2$

Assumption: only one polarization component is present!

In the rotating wave approximation

$$i\dot{a}_g = (-1)^{\mu+1}\hbar^{-1}(d_{eg}E_{-\mu})^* a_e$$

$$i a_e = -(\Delta+i\gamma)a_e + (-1)^{\mu+1}\hbar^{-1} d_{eg}E_{-\mu} a_g$$

If the e.m. field does not saturate the transition, i.e., if

$$\left|\frac{d_{eg}E_{-\mu}}{\hbar}\right| \ll \min(|\Delta|, \gamma)$$

then $a_e \approx \dfrac{(-1)^{\mu+1}}{\hbar(\Delta+i\gamma)} d_{eg} E_{-\mu} a_g$

Adiabatic elimination of the excited state

Energy shift and width of the ground state irradiated by resonant light

$$\dot{a}_g = (-i\,\Delta\omega_{Stark} - W)\,a_g$$

a.c. Stark shift

$$\Delta\omega_{Stark} \equiv \frac{\Delta E_{stark}}{\hbar} = \frac{\Delta}{\Delta^2+\gamma^2}\left|\frac{d_{eg}E_{-\mu}}{\hbar}\right|^2$$

Optically induced width of $|g\rangle$ (imaginary correction to the energy, divided by $\hbar$):

$$W = \frac{\gamma}{\Delta^2+\gamma^2}\left|\frac{d_{eg}E_{-\mu}}{\hbar}\right|^2$$

$$|a_g(t)|^2 = |a_g(0)|^2 \exp(-2Wt)$$

$2W$ = decay rate of the state $|g\rangle$ =

= photon scattering rate

Absorption cross-section, by definition,

$$\sigma = \frac{\text{Scattered energy}}{\text{Intensity}} = \frac{2W\hbar\omega}{I}$$

$$\sigma = \frac{1}{1+(\Delta/\gamma)^2}\,\sigma_0 ,$$

where the cross-section exactly at resonance ($\Delta = 0$) is

$$\sigma_0 = 2\hbar\omega\left|\frac{d_{eg}E_{-\mu}}{\hbar}\right|^2 \Big/ \left(\gamma \cdot 2c\varepsilon_0|E_{-\mu}|^2\right) =$$

$$= \frac{|d_{eg}|^2\,\omega}{\hbar c\varepsilon_0\gamma} \approx \frac{|d_{eg}|^2\,\omega_{eg}}{\hbar c\varepsilon_0\gamma}$$

Partial width (corresponding to the radiative decay $|e\rangle \to |g\rangle$):

$$\gamma_{eg} = \frac{|d_{eg}|^2}{6\pi\varepsilon_0\hbar}\left(\frac{\omega_{eg}}{c}\right)^3$$

γ = Sum of all partial width (sum over all final states of radiative decay)

$$\sigma_0 = \frac{3}{2\pi}\lambda^2\,\frac{\gamma_{eg}}{\gamma} ,$$

where $\lambda = \dfrac{2\pi c}{\omega_{eg}}$ = wavelength of resonant radiation

The maximum possible value of $\sigma = \frac{3}{2\pi}\lambda^2$ is obtained for alkali atoms on a cycling transition in D_2-line, where the optically excited state has only one decay channel.

For 87 Rb :

$$\underbrace{|F_g = 2, M_g = 2\rangle}_{\text{ground state}} \xrightarrow[\sigma_+]{} \underbrace{|F_e = 3, M_g = 3\rangle}_{\text{upper fine component of the first excited state}}$$

In a case of D_1- or D_2- line (when $|e\rangle$ is the first electronic excited state, that decays to sublevels of the ground state only)

$$\frac{\gamma_{eg}}{\gamma} = |C^{F_e M_e}_{F_g M_g 1\mu}|^2 (2L_e + 1) \times$$

$$\times (2J_e + 1)(2J_g + 1)(2F_g + 1) \times$$

$$\times \begin{Bmatrix} J_e & I & F_e \\ F_g & 1 & J_g \end{Bmatrix}^2 \begin{Bmatrix} L_e & S & J_e \\ J_g & 1 & L_g \end{Bmatrix}^2$$

Here we recalled that, after all summations

$$\gamma = \frac{|\langle n_g L_g \| \hat{d} \| n_e L_e \rangle|^2}{6\pi\varepsilon_0 \hbar (2L_e + 1)} \left(\frac{\omega_{eg}}{c}\right)^3$$

Stark shift

a.c. Stark shift : single resonant level

$$\omega_{Stark} = \frac{\Delta}{\gamma} W = \frac{\Delta}{\gamma} \frac{\sigma I}{2\hbar\omega}$$

$$\sigma = \frac{1}{1+(\Delta/\gamma)^2} \sigma_0 = \frac{1}{1+(\Delta/\gamma)^2} \frac{3\lambda^2}{2\pi} \frac{\gamma_{eg}}{\gamma} =$$

$$= \frac{1}{1+(\Delta/\gamma)^2} \sigma_{max} \frac{\gamma_{eg}}{\gamma}$$

In the practically important case $|\Delta| \gg \gamma$ (but we still consider Δ that is small compared to the hyperfine splitting of the excited state)

We have

$$\omega_{Stark} \approx \frac{\gamma}{\Delta} \frac{\gamma_{eg}}{\gamma} \frac{\sigma_{max} I}{2\hbar\omega}$$

All the state/polarization dependence is in

$$\frac{\gamma_{eg}}{\gamma} = \left| C^{F_e M_e}_{F_g M_g 1 \mu} \right|^2 (2L_e+1)(2J_e+1)(2J_g+1)(2F_g+1) \times$$

$$\times \begin{Bmatrix} J_e & I & F_e \\ F_g & 1 & J_g \end{Bmatrix}^2 \begin{Bmatrix} L_e & S & J_e \\ J_g & 1 & L_g \end{Bmatrix}^2$$

We see from the analytic expressions for $C^{F_e M_e}_{F_g M_g 1 \mu}$, that the squares of these C.-G.-coefficients depend on M_g quadratically:

$$\left| C^{F_e M_e}_{F_g M_g 1 \mu} \right|^2 = \alpha_0 + \alpha_1 M_g + \alpha_2 M_g^2$$

What is the (mathematical) physics behind?

$$\omega_{Stark} = \left(C^{F_e M_e}_{F_g M_g 1 \mu} \right)^2 \tilde{\omega}_0$$

$\tilde{\omega}_0 \propto I$ = what remains after factorizing out $\left(C^{F_e M_e}_{F_g M_g 1 \mu} \right)^2$.

Calculating ω_{Stark} for $|F_g M_g\rangle$ sublevel of the ground state and the known polarization μ, we can formally add a sum:

$$\omega_{Stark} = \sum_{M_e} \left(C^{F_e M_e}_{F_g M_g 1 \mu} \right)^2 \tilde{\omega}_0 ,$$

where only the term with $M_e = M_g + \mu$ contributes

$$C^{F_e M_e}_{F_g M_g\, 1\mu} = (-1)^{1+\mu}\sqrt{\frac{2F_e+1}{2F_g+1}}\, C^{F_g M_g}_{1-\mu\, F_e M_e} =$$

$$= (-1)^{F_e-F_g+\mu}\sqrt{\frac{2F_e+1}{2F_g+1}}\, C^{F_g M_g}_{F_e M_e\, 1-\mu}$$

$$\omega_{Stark} = \sum_{M_e} (-1)^{F_e-F_g+\mu}\sqrt{\frac{2F_e+1}{2F_g+1}} \times$$

$$\times C^{F_g M_g}_{F_e M_e\, 1-\mu}\, C^{F_e M_e}_{F_g M_g\, 1\mu}\, \tilde{\omega}_0$$

Consider the direction vector

$$\vec{n} = \sin\theta\cos\varphi\, \vec{e}_x + \sin\theta\sin\varphi\, \vec{e}_y + \cos\theta\, \vec{e}_z$$

and its cyclic components as an irreducible tensor of rank 1

Analogously to matrix elements of the dipole moment operator (actually, for a single outer electron $\vec{d} = -er\vec{n}$), we obtain

$$\langle F_e M_e | n_\mu | F_g M_g \rangle = (-1)^{L_e+S+J_e+J_g+I+F_g} \times$$

$$\times C^{F_e M_e}_{F_g M_g\, 1\mu}\sqrt{(2J_e+1)(2J_g+1)(2F_g+1)} \times$$

$$\times \begin{Bmatrix} J_g & I & F_g \\ F_e & 1 & J_e \end{Bmatrix}\begin{Bmatrix} L_g & S & J_g \\ J_e & 1 & L_e \end{Bmatrix}\langle L_e \| n \| L_g \rangle$$

It is known that

$$\langle L_e \| n \| L_g \rangle = \sqrt{2L_g+1}\, C^{L_e 0}_{L_g 0\, 10}$$

Recall also

$$\begin{Bmatrix} J_g & I & F_g \\ F_e & 1 & J_e \end{Bmatrix} = \begin{Bmatrix} J_g & 1 & J_e \\ F_e & I & F_g \end{Bmatrix} =$$

$$= \begin{Bmatrix} F_e & I & J_e \\ J_g & 1 & F_g \end{Bmatrix} = \begin{Bmatrix} F_e & 1 & F_g \\ J_g & I & J_e \end{Bmatrix}$$

and analogous relations for

$$\begin{Bmatrix} L_g & S & J_g \\ J_e & 1 & L_e \end{Bmatrix}$$

Recall the angular-momentum dependence of $\tilde{\omega}_0 = \frac{\gamma_{eg}}{\gamma} \tilde{\omega}_{sc}$,

where $\tilde{\omega}_{sc}$ does not depend on F_e, F_g etc (Scalar):

$$\omega_{Stark} = (-1)^{\mu} \sum_{M_e} \langle F_g M_g | n_{-\mu} | F_e M_e \rangle \langle F_e M_e | n_{\mu} | F_g M_g \rangle$$

$$\times \frac{(2L_e+1)(-1)^{L_e-L_g}}{\sqrt{(2L_e+1)(2L_g+1)}\, C^{L_e 0}_{L_g 0\, 1 0}\, C^{L_g 0}_{L_e 0\, 1 0}} \tilde{\omega}_{sc}$$

$$C^{L_e 0}_{L_g 0 1 0} = (-1)^{L_e - L_g} \sqrt{\frac{2L_e+1}{2L_g+1}}\, C^{L_g 0}_{L_e 0 1 0}$$

$$\omega_{Stark} = (-1)^{\mu} \sum_{M_e} \langle F_g M_g | n_{-\mu} | F_e M_e \rangle \langle F_e M_e | n_{\mu} | F_g M_g \rangle$$

$$\times \tilde{\omega}_{sc} \Big/ |C^{L_g 0}_{L_e 0 1 0}|^2$$

$\sum_{M_e=-F_e}^{F_e} |F_e M_e\rangle\langle F_e M_e|$ is a scalar

(IR tensor of rank 0).

$$\omega_{Stark} = (-1)^{\mu} \langle F_g M_g | P_{-\mu} n_{\mu} | F_g M_g \rangle \frac{\tilde{\omega}_{sc}}{|C^{L_g 0}_{L_e 0 1 0}|^2}$$

$$P_{-\mu} = n_{-\mu} \sum_{M_e=-F_e}^{F_e} |F_e M_e\rangle\langle F_e M_e|$$

$$P_{\mu'} n_{\mu} = \sum_{k=0}^{2} C^{k \mu'+\mu}_{1\mu'\, 1\mu} \{P \otimes n\}_{k \mu'+\mu}$$

$$\{P \otimes \mathbf{n}\}_{k \mu'+\mu} = \sum_{\mu_1 \mu_2} C^{k\mu'+\mu}_{1\mu_1\, 1\mu_2} P_{\mu_1} n_{\mu_2}$$

Therefore

$$\omega_{Stark} = \sum_{k=0}^{2} (-1)^{F_e - F_g + \mu} C^{k0}_{1-\mu\, 1\mu} \sqrt{\frac{2F_e+1}{2F_g+1}} \times$$

$$\times \sum_{M_e \mu_1 \mu_2} C^{k0}_{1\mu_1\, 1\mu_2} C^{F_g M_g}_{F_e M_e 1 \mu_1} C^{F_e M_e}_{F_g M_g 1 \mu_2} \tilde{\omega}_0 =$$

$$= \sum_{k=0}^{2} (-1)^{F_e+F_g+k+\mu} C^{F_g M_g}_{F_g M_g\, k0} C^{k0}_{1-\mu 1\mu} \times$$

$$\times \sqrt{\frac{2k+1}{2F_g+1}} (2F_e+1) \begin{Bmatrix} 1 & 1 & k \\ F_g & F_g & F_e \end{Bmatrix} \tilde{\omega}_0$$

$$\begin{Bmatrix} 1 & 1 & k \\ F_g & F_g & F_e \end{Bmatrix} = \begin{Bmatrix} F_g & F_g & k \\ 1 & 1 & F_e \end{Bmatrix}$$

F_g, F_g, k must obey the triangle rule.

$k=0$ component is always present

$k=1$; for $F_g \geqslant \frac{1}{2}$

$k=2$: for $F_g \geqslant 1$

$$C^{10}_{10\,10} = 0$$

Hence, $k=1$ component is present for $\mu = \pm 1$ only.

$k=0$ gives scalar polarizability (no dependence on M_g)

$k=1$ gives vector polarizability ($\propto M_g$)

$k=2$ gives tensor polarizability $\left(\propto 3M_g^2 - F_g(F_g+1)\right)$

Consider in more detail the case of $k=0$

$$C^{00}_{1-\mu\,1\,\mu} = \frac{(-1)^{1-\mu}}{\sqrt{3}}, \quad C^{F_gM_g}_{F_gM_g00} = 1$$

$$\begin{Bmatrix} 1 & 1 & 0 \\ F_g & F_g & F_e \end{Bmatrix} = \frac{(-1)^{1+F_e+F_g}}{\sqrt{3(2F_g+1)}}$$

$F_e + F_g$ is integer, therefore

$$(-1)^{F_e+F_g+\mu} C^{F_gM_g}_{F_gM_g00} C^{00}_{1-\mu\,1\,\mu} \times \sqrt{\frac{1}{2F_g+1}}\,(2F_e+1) \begin{Bmatrix} 1 & 1 & 0 \\ F_g & F_g & F_e \end{Bmatrix} \tilde{\omega}_0 =$$

$$= \frac{2F_e+1}{3(2F_g+1)}\tilde{\omega}_0$$

has the same sign as $\tilde{\omega}_0$, i.e., the same sign as Δ.

A general case

Polarizabilities (**u** the e.m.field polarization unit vector):

- Scalar (*k*=0) const
- Vector (*k*=1) $\propto [\mathbf{u}^* \times \mathbf{u}]\hat{\mathbf{J}}$
- Tensor (*k*=2) $\propto \left[(\mathbf{u}\hat{\mathbf{J}})(\mathbf{u}^*\hat{\mathbf{J}}) + (\mathbf{u}^*\hat{\mathbf{J}})(\mathbf{u}\hat{\mathbf{J}}) - \frac{2}{3}\hat{\mathbf{J}}^2\right]$

See, e.g., Fam Le Kien,
Scheeweiß & Rauschenbeutel,
PRA **88**, 033840 (2013)

Prefactors?

If $|\Delta|$ is much larger than the hyperfine splitting of the upper state, this detuning may be considered as (approximately) equal for all F_e's for a given fine-structure level. Then we can sum up over F_e.

$$\omega_{Stark} = \sum_{F_e M_e} (-1)^{\mu} \langle F_g M_g | n_{-\mu} | F_e M_e \rangle \langle F_e M_e | n_{\mu} | F_g M_g \rangle \times \frac{\widetilde{\omega_{SC}}}{|C^{L_g 0}_{L_e 0\, 1 0}|^2}$$

$$|F_e M_e\rangle = \sum_{M_{J_e} M_I} C^{F_e M_e}_{J_e M_{J_e} I M_I} |J_e M_{J_e}\rangle |I M_I\rangle$$

$$|F_g M_g\rangle = \sum_{M_{J_g} M_I} C^{F_g M_g}_{J_g M_{J_g} I M_I} |J_g M_{J_g}\rangle |I M_I\rangle$$

$\hat{n}_\mu$ does not act on nuclear-spin degrees of freedom.

Omitting the details,

$$\omega_{Stark} = \sum_{k=0}^{2} \omega^{(k)}_{Stark}, \quad \text{where}$$

$$\omega^{(k)}_{Stark} \propto C^{F_g M_g}_{F_g M_g k 0} \begin{Bmatrix} J_g & J_g & k \\ F_g & F_g & I \end{Bmatrix} \begin{Bmatrix} 1 & 1 & k \\ J_g & J_g & J_e \end{Bmatrix}$$

This means that not only F_g, F_g, k, but also J_g, J_g, k must satisfy the triangle rule. This means that for alkali atoms ($J_g = 1/2$) and detunings much larger than the hyperfine splitting of the 1st excited state ($|\Delta| \gg 2\pi \cdot 1$ GHz) only $k=0$ and $k=1$ give non-zero shift (Scalar and vector polarizability)

If the absolute value of the detuning is even much larger than the fine splitting of the excited state ($|\Delta| >> 2\pi \cdot 10$ THz), then

$$\omega_{\mathrm{Stark}}^{(k)} \propto \begin{Bmatrix} 1 & 1 & k \\ L_g & L_g & L_e \end{Bmatrix} = \begin{Bmatrix} L_g & L_g & k \\ 1 & 1 & L_e \end{Bmatrix}$$

This means that L_g, L_g, k must satisfy the triangle rule. For alkali atoms ($L_g = 0$) in this regime the Stark shift is characterized by scalar polarizability ($k = 0$) only.

In particular, the d.c. Stark shift (in a static electric field, $\omega = 0$) is always scalar ($k = 0$).

However, the static polarizability, unlike the near-resonant case, is determined by the size of the atom.