Macroscopic quantum tunnelling

- Regimes of Tunneling of interacting quantum gas
  Pendulum analogy
- Experimental techniques:
  How to create a ‘weak link’, observational tools
- Bosonic weak Link: experiment
- Noise thermometry
- Squeezing and entanglement
- Dynamics and phase-locking

tunnelling of an interacting gas: Josephson tunnelling

For the **symmetric** double well (ΔE=0) we find:

$$\boxed{\begin{aligned}\dot z(t) &= -\sqrt{1-z^2(t)}\,\sin\phi(t)\\ \dot\phi(t) &= \Lambda z(t) + \frac{z(t)}{\sqrt{1-z^2(t)}}\cos\phi(t)\end{aligned}}$$

two coupled differential equations describe the dynamics of the system

$$z = \frac{N_L - N_R}{N_L + N_R}$$

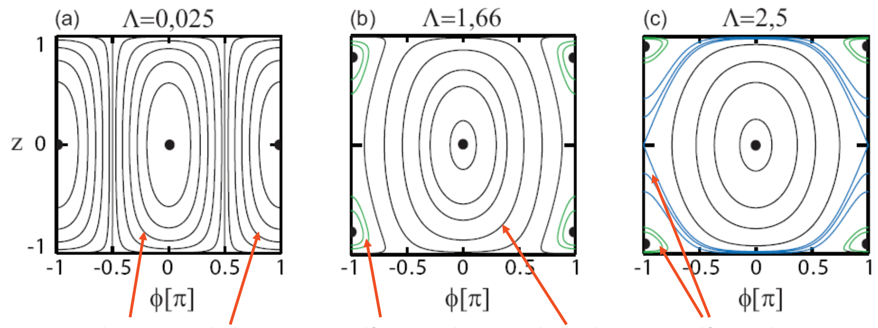
the parameter Λ (interaction energy / coupling energy) distiguishes several dynamical regimes:

Λ<<1 : **Rabi regime** - single particle behaviour, $\hbar\delta = 2K$
1< Λ<2 : **Josephson regime I** - macroscopic quantum self trapping for modes of $\langle\phi\rangle_t = \pi$ (pi-phase modes)
Λ>2 : **Josephson regime II** - macroscopic quantum self trapping for all modes, new running phase modes

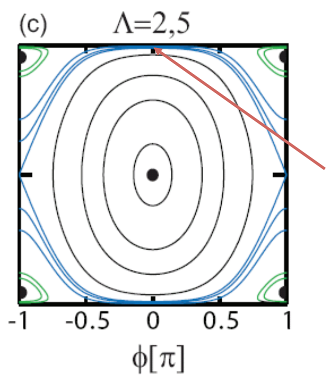
Josephson tunnelling

What are all these modes?

Rabi regime | Josephson I regime | Josephson II regime



The Josephson regime



In most experimental realizations one will always be in the Josephson regime ($\Lambda\approx 20$) (interaction energy > coupling energy) and it will be hard to observe oscillations

For ϕ(0)=0, the critical imbalance is

$$z_c = \frac{2}{\Lambda}\sqrt{\Lambda - 1} \qquad \text{(for } \Lambda=10 \text{ on finds } z_c=0.6\text{)}$$

low amplitude Josephson oscillations can be observed at the **plasma frequency**

$$\omega_{PL} = \frac{2K}{\hbar}\sqrt{\Lambda + 1}$$

the plasma frequency can be significantly higher than the single particle tunnelling frequency

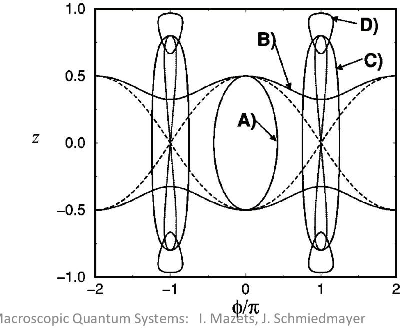
Josephson regime
pendulum analogy

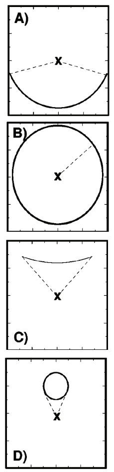
A mechanical analogon: The „momentum-shortened, non-rigid pendulum“

$$\dot{z}(t) = -\sqrt{1-z^2(t)}\,\sin\phi(t)$$

$$\dot{\phi}(t) = \Lambda z(t) + \frac{z(t)}{\sqrt{1-z^2(t)}}\cos\phi(t)$$

pendulum momentum: population imbalance
pendulum angle: phase difference





A) Josephson plasma oszillations (zero phase modes) (Λ=5)

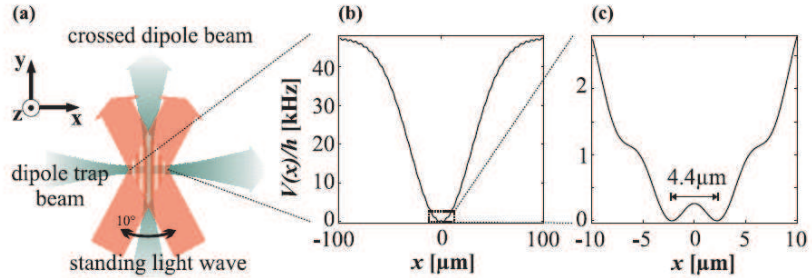
B) Running phase self trapped modes (Λ=25)

C) pi-phase oscillations without self trapping (Λ=0.36)

D) pi-phase oscillations with self trapping (Λ=2)

How to create a weak link Josephson junction

An all-optical douple well potential:
crossed dipole beam trap plus standing wave

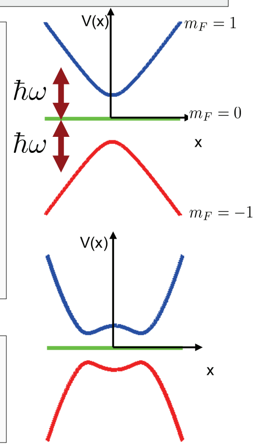


RF and MW induced adiabatic potentials

create adiabatic dressed state potentials by coupling electronic ground states of an atom

- coupling between stable states allows to create conservative potentials even with on resonant radiation
- shaping the potential:
  - detuning the states with an external magnetic field
  - spatial dependent coupling strength (RF field)

  -> allows strong field seeker traps
- coupling is magnetic:
  the amplitude and the relative orientation of the RF field and the detuning field are important

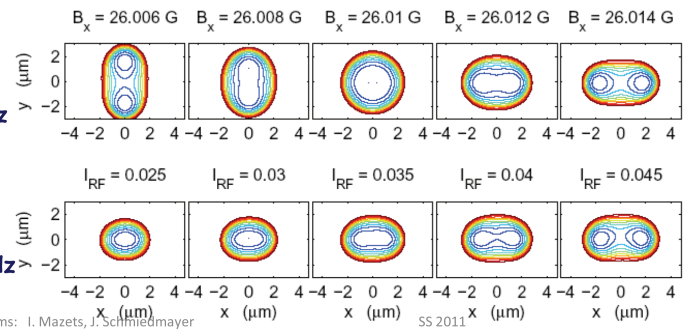


- first experiment: dressed neutrons: E. Muskat et al., PRL **58**, 2047 (1987).
- first proposal of a MW trap (detuned) C. Agosta, et al. PRL. **62**, 2361 (1989).
- MW experiment (Cs, detuned) R. Spreeuw, et al. PRL **72**, 3162 (1994).
- RF dressed state traps O. Zobay, B. M. Garraway, PRL **86**, 1195 (2001).
  (with magnetic field detuning but neglecting polarization)
- RF potentials for thermal Rb atoms: Y. Colombe, et al. Europhys. Lett. **67**, 593 (2004).
- Full implementation T. Schumm et al Nature Physics **1**, 57 (2005)

Advantages of RF potentials
splitting a trap

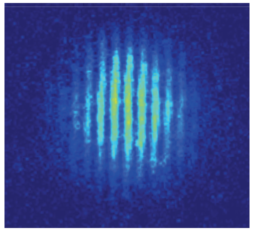
Hoffererth et al. Naure Physics 2, 710 (2006)

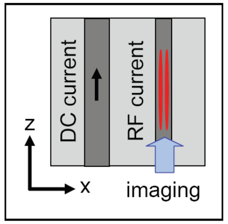
- True spliting 1 trap -> 2 traps
  - Confinement in transvrsal direction stays the same
  - Confinement in splitting direction is significantly tighter
- splitting potential: $V(x)=A(t)\,x^2+\mathbf{B}\,x^4$
  the size of the x^4 term determines the confinement
  In RF potentials **B** is factor ~1000 larger

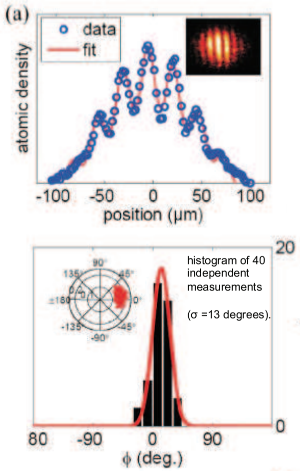


Coherent Splitting

'Matter-wave interferometry in a double well on an atom chip', T. Schumm, et al., Nature Physics. **1**, 57 (2005)

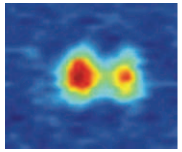




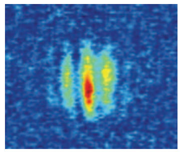


After the BECs has been split far enough to inhibit tunneling (d=3.4 μm), atoms are released and an interference pattern is observed after time of flight.

Observational Tools



in situ image of a BEC in a double well:
- 1100 atoms (reduce interactions)
- 1 μm spatial resolution



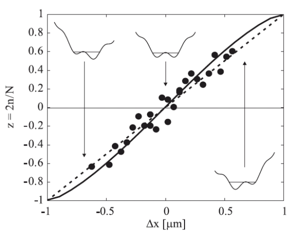
Interference image after time-of-flight

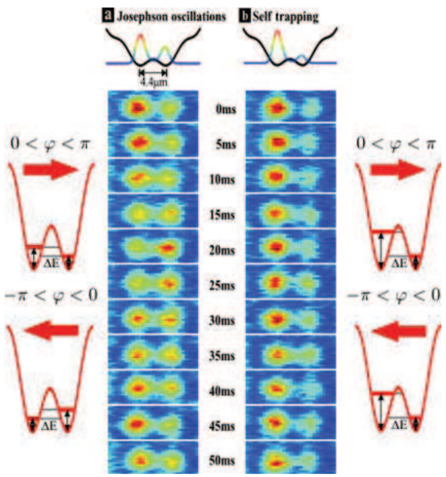
Number imbalance and relative phase of dynamics can be directly observed (strong contrast to superconductors or superfluids)

the bosonic weak link Josephson junction - experiment

Albiez et al. PRL 95, 010402 (2005)

Population imbalances can be realized by shifting the 1D optical lattice with respect to the optical dipole trap

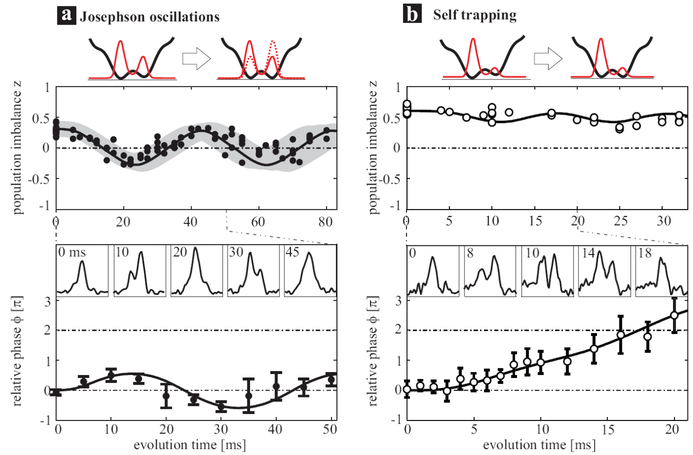




Starting with an imbalanced system and rapidly switching to the ballanced double well initiated dnamics

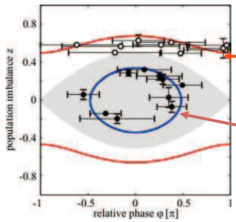
the bosonic weak link Josephson junction - experiment

Albiez et al. PRL 95, 010402 (2005)



the bosonic weak link Josephson junction - experiment

Albiez et al. PRL 95, 010402 (2005)



running phase
self trapped mode

Josephson plasma mode

Experimental parameters:

plasma frequency: 25 Hz
single particle δ: 2 Hz
Λ = 15(3) (deep in Josephson regime)
z_c = 0.50(5)

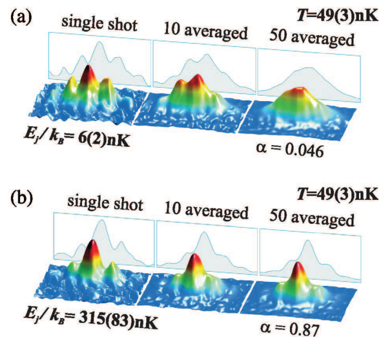
What determines the error on phase and atom number (error seems to be much bigger on the phase)

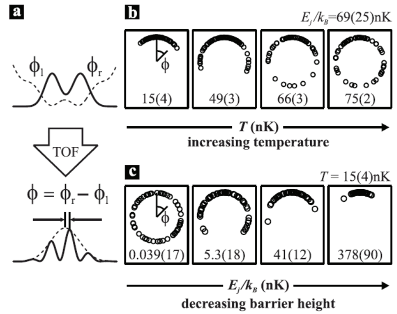
This is a temperature effect, similar to collective excitations in 3D or phase fluctuations in 1D and 2D

the bosonic weak link Josephson junction – Noise Thermometry

Gatti et al. PRL 96, 130404(2006)

measurements of the double well ground state in equilibrium (no imbalance)





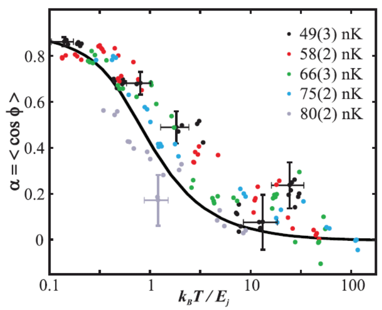
We characterize fluctuations of the relative phase by a **coherence factor**: where E_j describes the coupling energy

$$\alpha = \langle \cos\phi \rangle = \frac{\int_{-\pi}^{\pi} d\phi \cos\phi \exp(E_j / k_B T \cos\phi)}{\int_{-\pi}^{\pi} d\phi \exp(E_j / k_B T \cos\phi)}$$

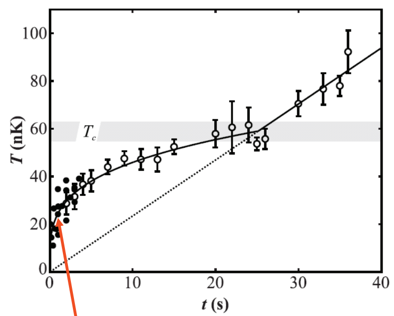
the bosonic weak link Josephson junction – Noise Thermometry

Gatti et al. PRL 96, 130404(2006)

Temperature dependence of the coherence factor



Monitoring technical heating in the experiment (40 nK/s)

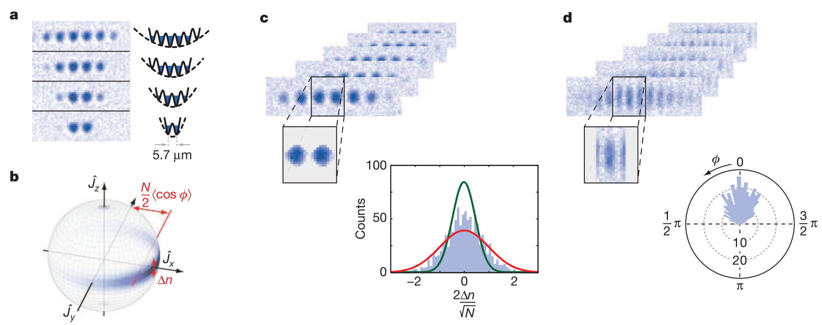


Noise thermometry in a double well is one of the most precise temperature measurements below T_c so far

rapid increase of temperature due to vanishing heat capacity at T=0

Squeezing and entaglement

Esteve al. Nature 455, 1216 (2008)



Squeezing and entaglement

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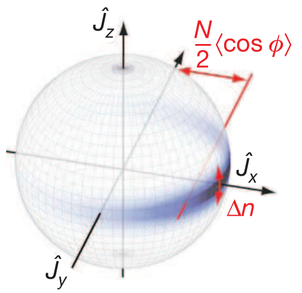
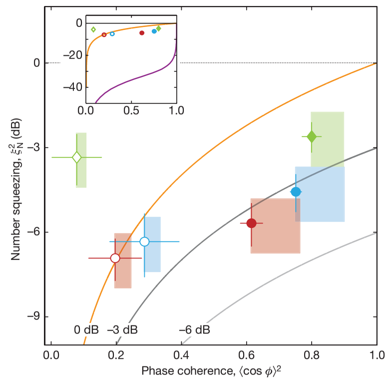


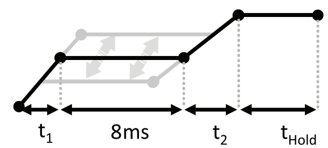


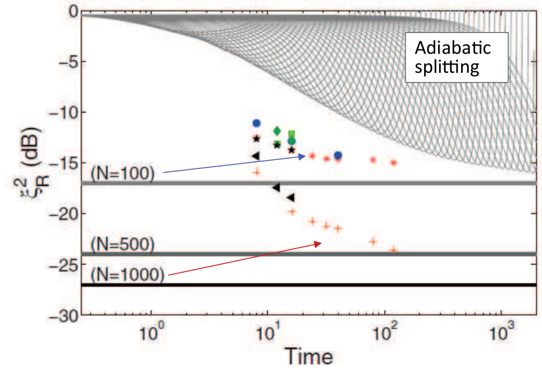
Optimal Control of Splitting
fast squeezing in a multi mode system

Optimal Controll applied to the problem of the fluctuation properties in splitting a BEC

J. Grond et al. PRA **79**, 021603 R (2009)
J. Grond et al. PRA **80**, 053625 (2009)

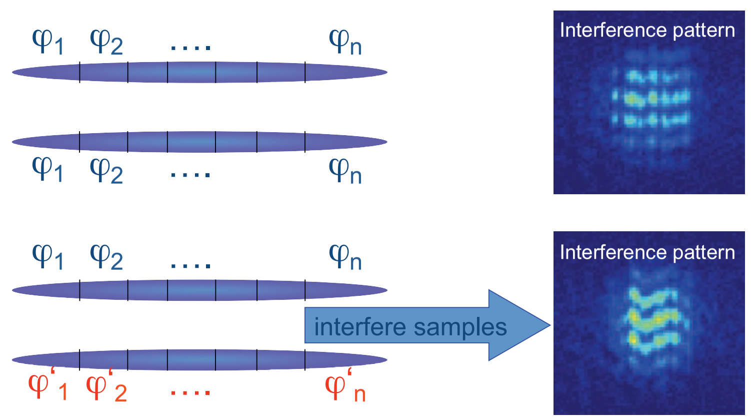
– Fancy splitting ramps inspired by OCT: t1+t2 = 17ms
– Leads to dramatic change of statistical distribution of interference





interference of phase fluctuating 1D condensates

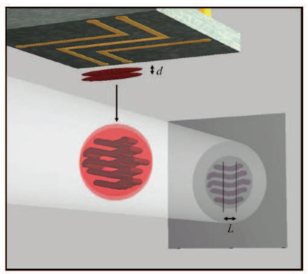
How can we study the dynamics of phase fluctuations?



Hofferberth et al Nature 449, 324 (2007)

Tunneling and Phaselocking

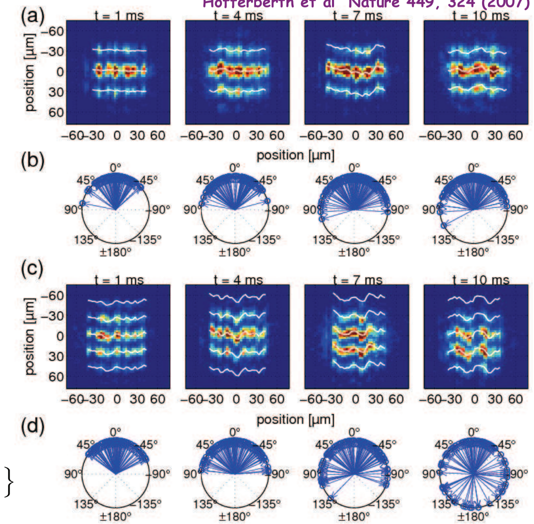
tunnel coupling keeps phase linked



coherence factor

$$\Psi = \frac{1}{L}\mathrm{Re}\left\{\int dz\,\left\langle e^{i(\phi_2(z)-\phi_1(z))}\right\rangle\right\}$$

Hofferberth et al Nature 449, 324 (2007)

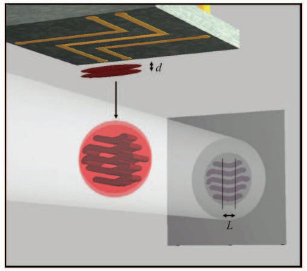


Tunneling and Phaselocking

Hofferberth et al Nature **449**, 324 (2007)

tunnel coupling:
phase locking

independent 1d Bose gas:
random phase



coherence factor

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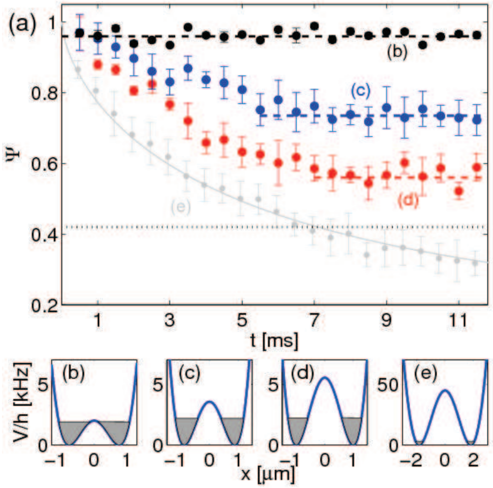
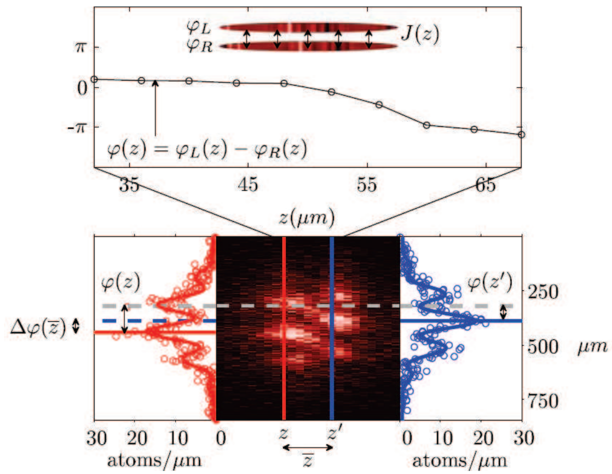


Fluctuating interference for coupled 1d systems

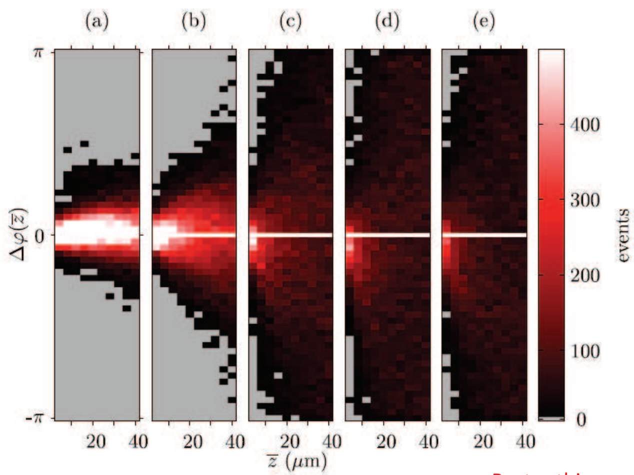
Experiment: T. Betz et al. PRL **106**, 020407 (2011)
Theory: Stimming et al. et al. PRL **105**, 015301 (2010)



Interference

relative phase change with distance

T. Betz et al. PRL **106**, 020407 (2011)

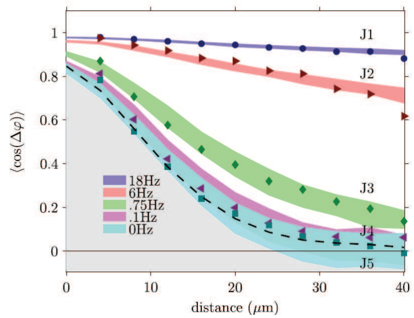


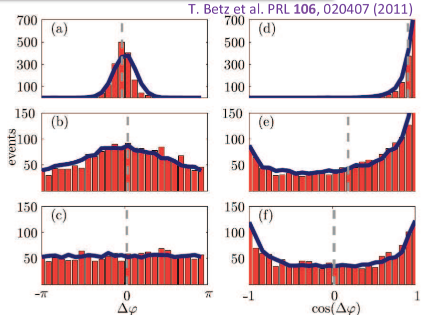
Poster this evening

Phase correlation

comparison to semi-classical model

T. Betz et al. PRL **106**, 020407 (2011)





$$l_J = \frac{1}{2}\sqrt{\frac{\hbar}{m_{Rb}J}}$$

$$\lambda_T = \frac{2\hbar^2 n_0}{m_{Rb}k_B T}$$

Setting	$T_{profile}$(nK)	T_{OU}(nK)	J_{OU}(Hz)	J_{simu}(Hz)	$\lambda_T(\mu m)$	$l_J(\mu m)$
J_1 T_0	154	-	16.5-21	35	5.26	2.95-3.33
J_2 T_0	150	-	5-7.5	4.8	4.88	4.93-6.04
J_3 T_0	153	-	0.65-0.88	0.12	4.76	14.40-16.76
J_4 T_0	154	-	0.05-0.15	0.04	4.54	34.88-60.41
J_5 T_0	163	-	<0.08	<0.04	4.34	>47.77

Phase locking for $l_J < \lambda_T$