Macroscopic quantum tunnelling

- Regimes of Tunneling of interacting quantum gas
- Pendulum analogy
- Experimental techniques:
  - How to create a 'weak link', observational tools
- Bosonic weak link: experiment
- Noise thermometry
- Squeezing and entanglement
- Dynamics and phase-locking

Josephson tunnelling

What are all these modes?

(a) $\Lambda=0.025$
(b) $\Lambda=1.66$
(c) $\Lambda=2.5$

- Rabi regime
- Josephson I regime
- Josephson II regime

The Josephson regime

In most experimental realizations one will always be in the Josephson regime ($\Lambda \approx 20$) (interaction energy > coupling energy) and it will be hard to observe oscillations

For $\phi(0)=0$, the critical imbalance is

$$z_c = \frac{2}{\Lambda} \sqrt{\Lambda - 1}$$

(for $\Lambda=10$ on finds $z_c=0.6$)

Low amplitude Josephson oscillations can be observed at the plasma frequency

$$\omega_{pl} = \frac{2K}{\hbar} \sqrt{\Lambda + 1}$$

The plasma frequency can be significantly higher than the single particle tunnelling frequency
A mechanical analogon: The "momentum-shortened, non-rigid pendulum"

\[
\\dot{z}(t) = -\sqrt{1-z^2(t)} \sin \phi(t)
\]

\[
\phi(t) = \dot{z}(t) + \sqrt{1-z^2(t)} \cos \phi(t)
\]

pendulum momentum: population imbalance
pendulum angle: phase difference

Josephson plasma oscillations (zero phase modes) \((\Lambda=5)\)
Running phase self trapped modes \((\Lambda=25)\)
Pi-phase oscillations without self trapping \((\Lambda=0.36)\)
Pi-phase oscillations with self trapping \((\Lambda=2)\)

Josephson regime pendulum analogy

How to create a weak link Josephson junction

An all-optical double well potential: crossed dipole beam trap plus standing wave

RF and MW induced adiabatic potentials

create adiabatic dressed state potentials by coupling electronic ground states of an atom

- coupling between stable states allows to create conservative potentials even with resonant radiation
- shaping the potential:
  - detuning the states with an external magnetic field
  - spatial dependent coupling strength (RF field)
  - allows strong field seeker traps
- coupling is magnetic:
  - the amplitude and the relative orientation of the RF field and the detuning field are important

Advantages of RF potentials splitting a trap

- True splitting 1 trap \(\rightarrow\) 2 traps
  - Confinement in transversal direction stays the same
  - Confinement in splitting direction is significantly tighter
- splitting potential: \(V(x) = A(t)x^2 + Bx^4\) the size of the \(x^4\) term determines the confinement
  - In RF potentials \(B\) is factor \(\sim 1000\) larger
After the BECs has been split far enough to inhibit tunneling ($d=3.4 \mu m$), atoms are released and an interference pattern is observed after time of flight.


In situ image of a BEC in a double well:
- 1100 atoms (reduce interactions)
- 1 \mu m spatial resolution

Interference image after time-of-flight

Population imbalances can be realized by shifting the 1D optical lattice with respect to the optical dipole trap starting with an imbalanced system and rapidly switching to the balanced double well initiated dynamics.

Starting with an imbalanced system and rapidly switching to the balanced double well initiated dynamics.
Experimental parameters:
- plasma frequency: 25 Hz
- single particle: 2 Hz
- $\Lambda = 15(3)$ (deep in Josephson regime)
- $z_c = 0.50(5)$

What determines the error on phase and atom number (error seems to be much bigger on the phase)

This is a temperature effect, similar to collective excitations in 3D or phase fluctuations in 1D and 2D.

We characterize fluctuations of the relative phase by a coherence factor:

$$\alpha = \frac{\int d\phi \cos \phi (E / k_b T \cos \phi)}{\int d\phi \sin \phi (E / k_b T \sin \phi)}$$

where $E$ describes the coupling energy.

Noise thermometry in a double well is one of the most precise temperature measurements below $T_c$ so far.

Rapid increase of temperature due to vanishing heat capacity at $T = 0$. 

Counts
How can we study the dynamics of phase fluctuations?

Interference pattern

$\Psi_1, \Psi_2, \ldots, \Psi_n$

Interference pattern

$\Psi'_1, \Psi'_2, \ldots, \Psi'_n$

$\Psi_1, \Psi_2, \ldots, \Psi_n$

$\Psi'_1, \Psi'_2, \ldots, \Psi'_n$

Tunneling and Phaselocking

$\Psi = \frac{1}{2} \text{Re} \left\{ \int dz \left( e^{i(\Psi(z) - \Psi'(z))} \right) \right\}$

Tunneling and Phaselocking

tunnel coupling:
phase locking
independent 1d Bose gas:
random phase

coherence factor
\[ \Psi = \frac{1}{2} \text{Re} \left\{ \int dz \langle e^{i(\phi_1(z) - \phi_2(z))} \rangle \right\} \]

Fluctuating interference for
coupled 1d systems

Interference
relative phase change with distance

Phase correlation
comparison to semi-classical model

Phase locking for \( l_j < \lambda_T \)