Macroscopic Quantum Systems

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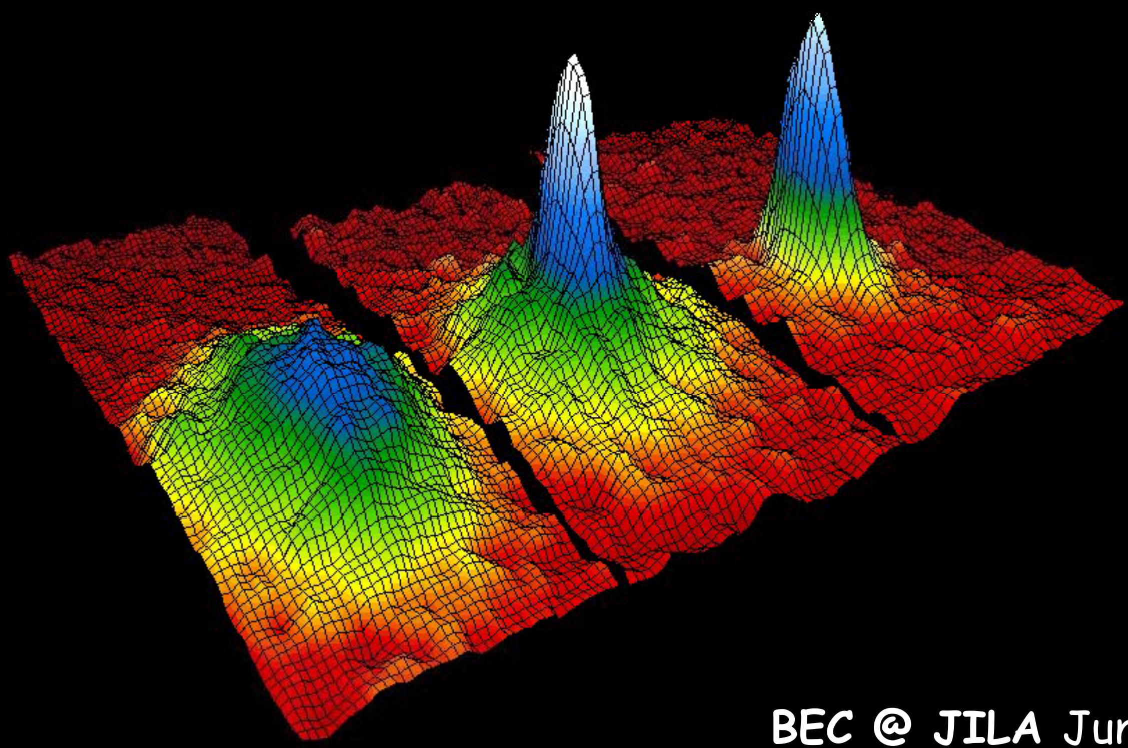


BEC @ JILA June '95

Course Outline

Macroscopic Quantum Systems 2011

02.03.	09.03.	14.-18. 03	06.04. tentative	27. 04. tentative	01. 06. tentative	08. 06. tentative
14^{15}-16^{30} Introduction to Bose-Einstein condensation Experimental techniques for achieving BEC	14^{15}-16^{30} Interacting quantum gases Basic Theory	Guest lectures on BEC Nick Proukakis Univ. Newcastle	14^{15}-17^{45} Dynamics of a Quantumgas Rotating Quantumgas Superfluidity and vortices	14^{15}-17^{45} Tunable Interactions Feshbach resonance Molecule formation Degenerate Fermi gases BEC-BCS crossover	14^{15}-17^{45} Low dimensional quantum gases Coherence properties of BEC	14^{15}-17^{45} Quantum coherence, quantum tunnelling Optical Lattices

There will be an extended coffee break in mid afternoon

Jörg Schmiedmayer	(experiment)
Igor Mazets	(theory)

Course Modalities

Seminars:

- one seminar session for each lecture block (2 in total)
- short 15+5 min. talks by small teams (up to 3 people)
- specific topics treated in more detail than in the lecture
- add experimental details
- practice for the exam (participation mandatory)
- Dates announced for each session

First seminar session: Wednesday **30th March 14h** (tentative)

Exam:

- short term-paper (4 pages max.) on a recent scientific publication
- individual topics defined, given out at last lecture block
- 72 h home exam, all tools (apart other people) allowed
- 15 min. discussion when handing in the paper

Lecture I:

1 Degenerate Quantum Gases: basic physics

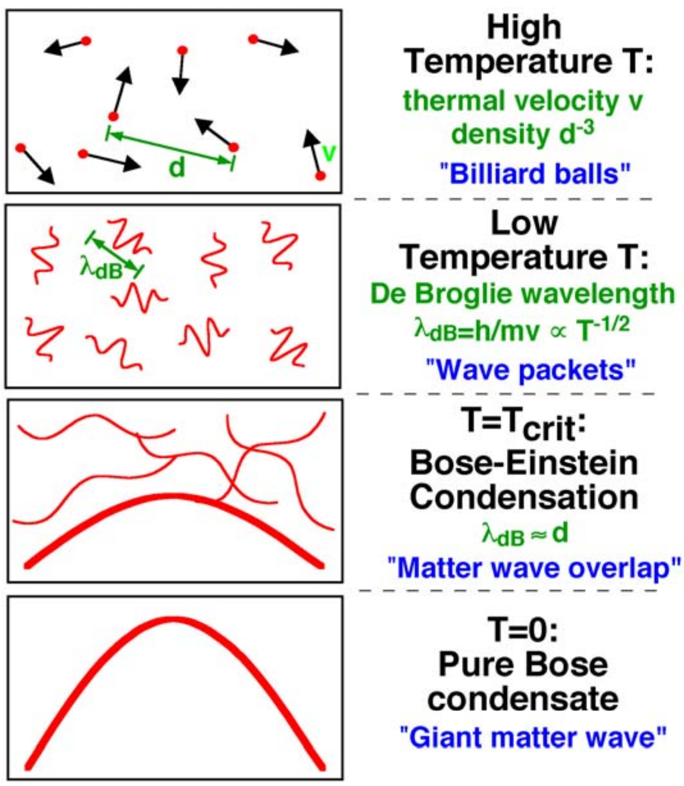
1.1 Quantum statistics:
- Wave function of 2 indistinquishable particles (Bosons – Fermions)
- Statistics and indistinguishability: Counting particle
- Density of states,
- Bose- Fermi and Bolzman Statistics (reminder)

1.2 Ideal Gas: BEC in 3d, degenerate Fermions
- Critical temperature for Bosons
- Condensate fraction

1.3 Ideal gas in a Trap
- Density of states in a trap
- BEC in a Trap
- Fermions in a Trap
- Lower dimensions

1.4 Making a BEC
- Laser Cooling
- Magnetic Trapping
- Evaporative Cooling
- observing a BEC

BEC
basic introduction



What is BEC? What is its underlying Physics? What the fundamental concept?

Colloquial: *'all particles are in the same state'*
- Broken Gauge Symmetry,
- Off-diagonal long range order (ODLRO)
- Long range phase coherence
- Macroscopic wave function of the condensate

These concepts were first introduced in studying superconductivity and superfluidity

What is the signature?
- Delta function of the occupation number of particles with zero momentum associated with long range phase coherence
- Bose narrowing (decrease in average energy as density gets higher). For fermions it is the opposite.
- Process of stimulated scattering: The scattering rate contains a factor $(1+N_f)$ where N_f is the occupation number of the final state

BEC
basic introduction

BEC is a common phenomenon occurring in physics on all scales

- Condensed matter
- atomic physics
- nuclear and elementary particle physics
- astrophysics

Bosonic degrees of freedom are composite, they originate from Fermionic degrees of freedom (in most cases)**.**

- Fundamental Bosons:
  gauge Bosons : Photon, W,Z
- Fundamental Fermions:
  p,n,e ….

old table from 1993: *Bosons under study*

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^{4}He	$^4He^{2+}, 2e^-$	He-II	superfluidity
^{3}He pairs	$2(^3He^{2+}, 2e^-)$	^{3}He-A,B phases	superfluidity
cesium	$^{133}Cs^{55+}, 55e^-$	laser traps	(in progress)
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

BEC
dilute gas

from Eric Cornell's Nobel Prize Lecture

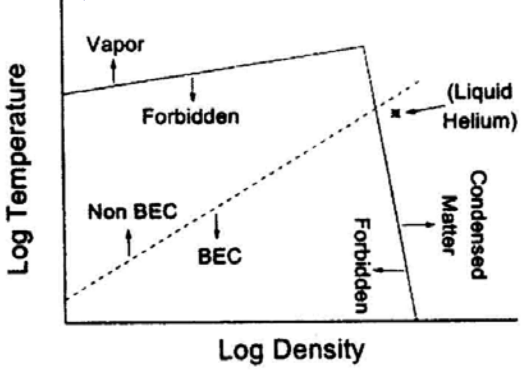


FIG. 1. Generic phase diagram common to all atoms: dotted line, the boundary between non-BEC and BEC; solid line, the boundary between allowed and forbidden regions of the temperature-density space. Note that at low and intermediate densities, BEC exists only in the thermodynamically forbidden regime.

Why interesting?

Strongly interacting vs. weakly interacting Bose gas

- Liquid Helium is dominated by interactions. The BEC fraction is in the order of 10%. Many phenomena are masked by the strong interactions
- A weakly interacting gas (Atoms, Excitons): theoretic description is easier
- using *Feshbach resonances* the interactions can be tuned by the experimenter: weakly interacting -> strongly correlated

Condensation in free space vs. trapped condensates

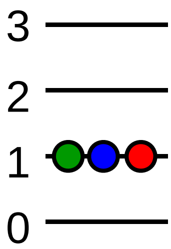
- Free space one gets the classic formulas for BEC and its thermodynamic properties.
- Trapped gases: one has to look at the density of states in the trap.
  - isotropy of trap potential
  - dimensionality: 3d, (quasi) 2d, 1d
  - disordered potentials
- lattices: probe (simulate) solid state problems
- small number of particles vs. continuum in thermodynamics
  - what is the minimal size of a system we still can call a Bose condensate?

Fermions

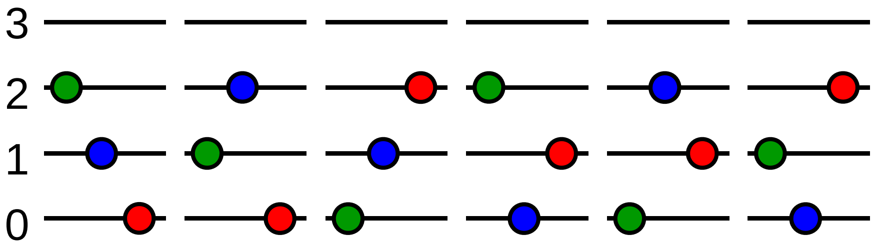
- Pauli principle, FD statistics
- At low temperatures: BEC vs. BCS
  - BEC: particle correlation length is very short compared to particle spacing. Molecules
  - BCS: particle correlation length is larger than the inter particle spacing, Cooper pairs

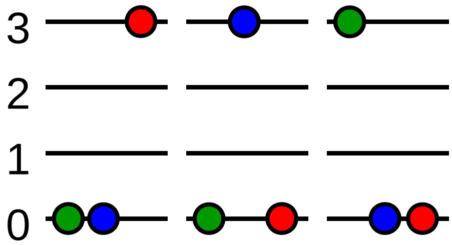
Counting Particles
classical distinguishable particles

3 particles,
total energy = 3



10 % probability
for triple occupancy



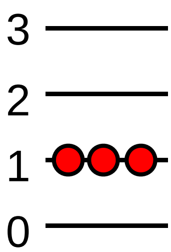


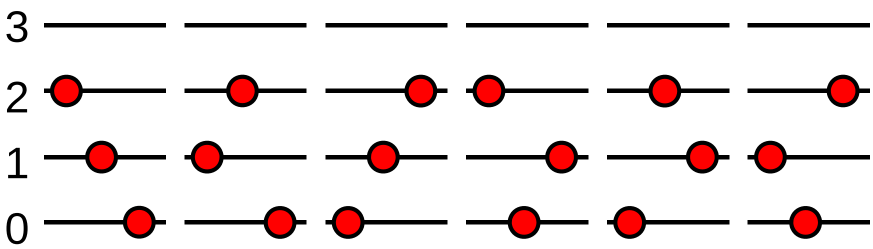
# Teilchen	Wahrscheinl.	wie oft **rot**
3	10%	1
6	20%	2
9	30%	3
12	40%	4

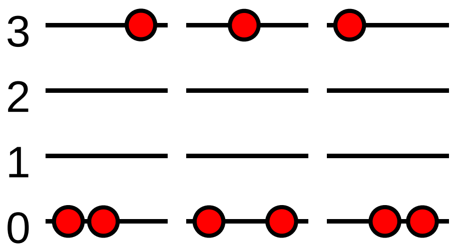
30 % probability
for double occupancy

Counting Particles
indistinguishable particles

3 particles,
total energy = 3

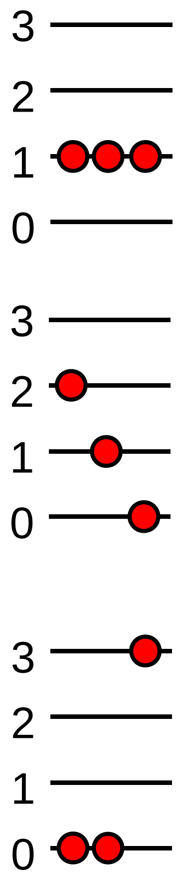






Counting Particles
Bosons

3 particles,
total energy = 3



33 % probability
for triple occupancy

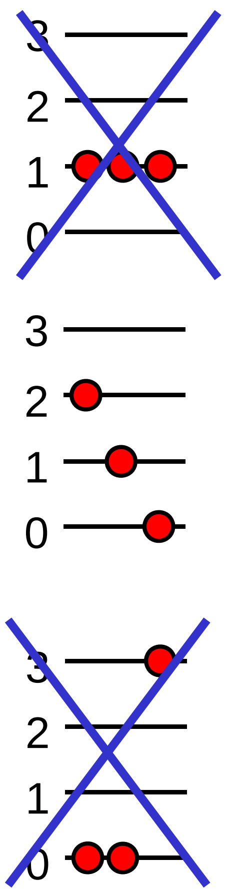
Bosons are gregarious!

1	11%
1	11%
4	44%
3	33%

33 % probability
for double occupancy

Counting
Fermions

3 particles,
total energy = 3



Fermions are loners!

100 % probability
for single occupancy

	fermions	bosons	classical
0	0%	11%	10%
1	33%	11%	20%
1	33%	44%	30%
1	33%	33%	40%

Quantenstatistik
3 verschieden Statistiken (Verteilungen)

	Bose	Boltzmann	Fermi
Teilchen	ununterscheidbar	unterscheidbar	ununterscheidbar
Spin	ganzzahlig $(0, 1, 2, \ldots)$	-	halbzahlig $(\frac{1}{2}, \frac{3}{2}, \ldots)$
Eigenfunktionen	symmetrisch	-	antisymmetrisch
qualitatives Verhalten	besetzen bevorzugt gleiche Zustände	-	Pauli-Prinzip: besetzen nie gleiche Zustände
P(E)	$\frac{1}{e^{(\alpha + E/kT)} - 1}$	$\frac{1}{e^{(\alpha + E/kT)}}$	$\frac{1}{e^{(\alpha + E/kT)} + 1}$
übliche Schreibweise	$\frac{1}{e^{(\alpha + E/kT)} - 1}$	$Ae^{-\frac{E}{kT}}$	$\frac{1}{e^{(E - E_F)/kT} + 1}$
Beispiele	Photonengas (Plancksches Gesetz) Phononengas flüssiges Helium Bose-Einstein-Kondensat	klassische Gase bei jeder Temperatur	Entartetes Elektronengas in Festkörper-, Atom-, Kern- und Astrophysik

Quantum Statistics: Two indistinguishable Particles

Wave function for 1 Particle $\psi(x_1)$

Wave function for 2 Particles $\psi(x_1, x_2)$

We can only observe $|\psi(x_1, x_2)|^2$

If the particles are indistinguishable we find: $|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$

There are 2 possibilities:

Boson : $\psi_+(x_1, x_2)$: $\psi_+(x_1, x_2) = +\psi_+(x_2, x_1)$

$$\psi_+(x_1, x_2) = \frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) + \psi_2(x_1)\psi_1(x_2)]$$

Fermion $\psi_-(x_1, x_2)$: $\psi_-(x_1, x_2) = -\psi_-(x_2, x_1)$

$$\psi_-(x_1, x_2) = \frac{1}{\sqrt{2}}[\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)]$$

Two indistinguishable Particles II

sets see what happens if the two particles are at the same location (state)
that is if $x_1 = x_2$

$\psi_+(x_1, x_1) \neq 0 \rightarrow$ Bosons can occupy the same state

$\psi_-(x_1, x_1) = 0 \rightarrow$ Fermions can not occupy the same state

consequently for 2 Bosons at the same location (in the same state) we find:

$$|\psi(x,x)|^2 = 2\,|\psi_1(x)\psi_2(x)|^2$$

probabilities to find n particles in the same state:

n classical particles $\quad P_n = (P_1)^n$

n Bosons $\quad P_n^{Boson} = n!(P_1)^n$

probability to add another Boson to a state with n Bosons

$$P_{n+1}^{Boson} = (n+1)P_1 P_n^{Boson}$$

stimulated scattering, stimulated emission

N - Particles

The above discussion generalizes readily to the case of N particles. Suppose we have N particles with quantum numbers n_1, n_2, ..., n_N. If the particles are bosons, they occupy a **totally symmetric state**, which is symmetric under the exchange of *any two* particle labels:

$$|n_1 n_2 \cdots n_N; S\rangle = \sqrt{\frac{\prod_j N_j!}{N!}} \sum_p |n_{p(1)}\rangle |n_{p(2)}\rangle \cdots |n_{p(N)}\rangle$$

Here, the sum is taken over all possible permutations p acting on N elements. The square root on the right hand side is a normalizing constant. The quantity N_j stands for the number of times each of the single-particle states appears in the N-particle state.

In the same vein, fermions occupy **totally antisymmetric states**

$$|n_1 n_2 \cdots n_N; A\rangle = \frac{1}{\sqrt{N!}} \sum_p \mathrm{sgn}(p) |n_{p(1)}\rangle |n_{p(2)}\rangle \cdots |n_{p(N)}\rangle$$

Here, sgn(p) is the signature of each permutation (i.e. +1 if p is composed of an even number of transpositions, and −1 if odd.) Note that we have omitted the $\Pi_j N_j$ term, because each single-particle state can appear only once in a fermionic state.

These states have been normalized so that

$$\langle n_1 n_2 \cdots n_N; S | n_1 n_2 \cdots n_N; S\rangle = 1, \qquad \langle n_1 n_2 \cdots n_N; A | n_1 n_2 \cdots n_N; A\rangle = 1.$$

Fermionen

1. Beispiel: **2 freie Elektronen**

$$\psi_a(1) = e^{ik_a x_1} \qquad |\psi_a|^2 = const.$$
$$\psi_b(2) = e^{ik_b x_2} \qquad |\psi_b|^2 = const.$$
$$\Rightarrow \quad |\psi_-(1,2)|^2 = 1 - cos[(k_a - k_b)(x_1 - x_2)] \quad \text{(symmetrisch in } \Delta x \text{ und } \Delta k\text{)}$$

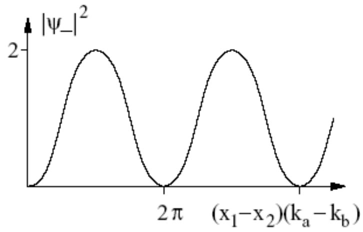
a) $x_1 = x_2$ $\Rightarrow$ $|\psi_-|^2 = 0$ überall

2 Elektronen können nie am gleichen Ort sein (gleiche Stelle im Ortsraum)

b) $k_a = k_b$ $\Rightarrow$ $|\psi_-|^2 = 0$ überall

2 Elektronen können nie den gleichen Impuls haben (gleiche Stelle im Impulsraum)

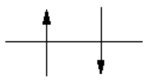
c) $k_a \neq k_b$



oszillatorische Wahrscheinlichkeitsdichte

Beachte: „Gleich" in Ort oder Impuls kann nur im Rahmen der Unschärferelation definiert werden, d.h. mit der Genauigkeit $\hbar$. Konsequenzen für die Quantenstatistik: „Zellgrößen" im Phasenraum sind von der Größenordnung des Planckschen Wirkungsquantums h (s. 3.2).

2. Beispiel: **Aufbau des Periodensystems**



Im 1s-Zustand von Atomen können gerade 2 Elektronen (mit entgegengesetzten Spinrichtungen) untergebracht werden, im 2s-Zustand ebenfalls 2, im 2p-Zustand 6 etc. (Vorschlag Pauli 1925)

$\Rightarrow$ Schalenstruktur der Atomhülle (s. Kap. 4)

Bosonen

P_1 sei Wahrscheinlichkeit, dass ein ursprünglich leerer Zustand von einem Boson besetzt wird. Bei n Bosonen gilt dann nicht wie bei klassischen (unterscheidbaren) Teilchen

$$P_n = (P_1)^n$$

sondern

$$\Rightarrow \quad P_n^{\text{Boson}} = n! P_1^n$$

Es ergibt sich also eine Vergrößerung der Besetzungswahrscheinlichkeit um den Faktor $n!$.

Differentielle Betrachtung:

$$P_{n+1}^{\text{Boson}} = (n+1) P_1 P_n^{\text{Boson}}$$

> Wenn schon n Bosonen in einem Zustand vorliegen, ist die Wahrscheinlichkeit, dass ein weiteres dazukommt, um $(n+1)$ größer als im Fall klassischer (unterscheidbarer) Teilchen.

Im Fall makroskopischer Systeme mit n von der Größenordnung $\geq 10^{20}$ führt dies zu enormen Konsequenzen (s.u.).

Merkregeln

- „Fermionen besetzen nie gleiche Zustände"
- „Bosonen bevorzugen gleiche Zustände"

Verteilung von Teilchen
Klassische und Quantensatistik

In der statistischen Mechanik gilt allgemein für die Zahl der Teilchen/Energieintervall dN/dE bei der Energie E

$$\frac{dN(E)}{dE} = g(E) \cdot P(E)$$

Dabei bedeuten

- $g(E)\,dE$ die Zahl der Zustände im Energieintervall dE bei der Energie E
- P(E) die Besetzungswahrscheinlichkeit (bzw. mittlere Zahl der Teilchen) von Zuständen der Energie E im thermischen Gleichgewicht

dN/dE kann verteilt sein

a) diskontinuierlich (gequantelte Zustände bei gebundenen Teilchen)
b) kontinuierlich (bei freien Teilchen)

Zustandsdichte

$g(E)$ muss im 6-dimensionalen Phasenraum (3 Orts-, 3 Impulskoordinaten) ermittelt werden.
Annahmen:

Ortsraum integral:	V
Impulsraum differentiell (Kugelschale):	$4\pi p^2 dp$
Größe einer Einzelzelle im Phasenraum:	h^3

Damit Zahl der Zustände im Impulsintervall dp beim Impuls p

$$g(p)dp = V\frac{4\pi p^2 dp}{h^3}$$

1) Nicht-relativistische Teilchen einschließlich Spin

$$p = \sqrt{2mE} \qquad \Rightarrow \qquad g(E)dE = g_s V \frac{4\pi}{h^3}(2m^3)^{1/2}\sqrt{E}dE$$

2) Voll-relativistische Teilchen (z.B. Photonen) einschließlich Spin

$$p = E/c \qquad \Rightarrow \qquad g(E)dE = g_s V \frac{4\pi}{h^3}\frac{E^2}{c^3}dE$$

Die Faktoren g_s bedeuten statistische Gewichte, die von der Größe des Spins abhängen

Derivation of the density of states for three dimensions

- Boundary condition: box
- Quantization of momentum
- Each point represents sphere with ħ^3 as volume

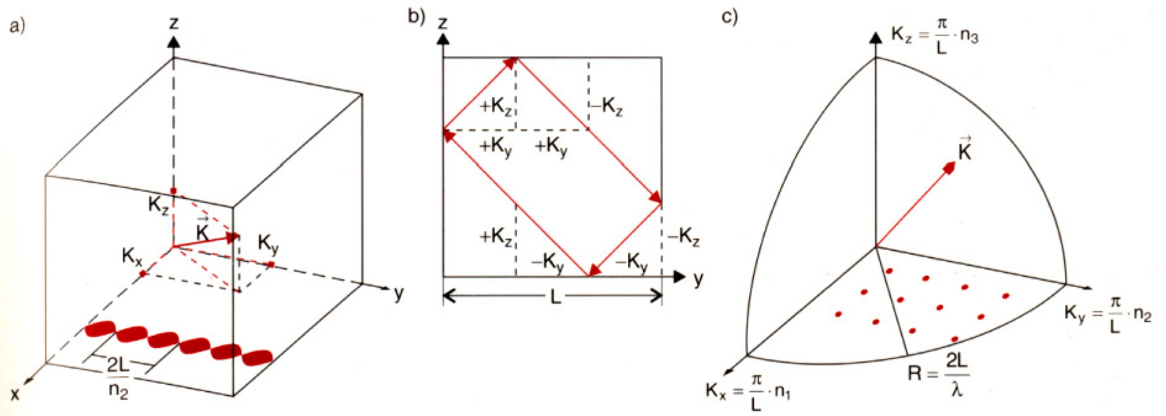


Abb. 12.10. (**a**) Stehende Wellen in einem Kasten. (**b**) Randbedingungen für die Wellenvektorkomponenten. (**c**) Zur Abzählung der Gitterpunkte im *K*-Raum

Quanten Statistik
Comparison of the Distributions

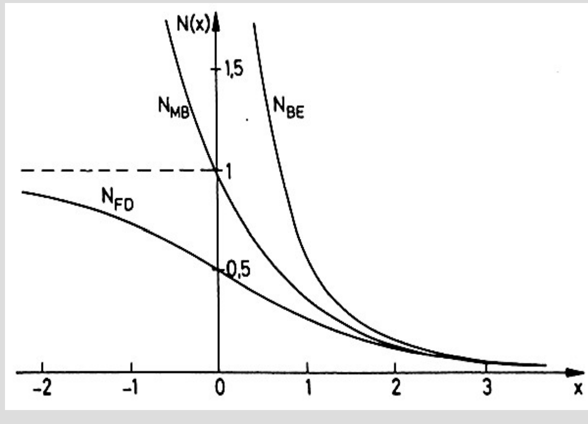
$$\sigma = e^{\frac{\mu}{kT}}, \quad x := \frac{\epsilon - \mu}{kT},$$

$$N_{iBE} = \frac{1}{\sigma^{-1} e^{\epsilon_i/kT} - 1} \longrightarrow N_{BE}(x) = \frac{1}{e^x - 1}$$

$$N_{iMB} = \frac{1}{\sigma^{-1} e^{\epsilon_i/kT}} \longrightarrow N_{MB}(x) = \frac{1}{e^x}$$

$$N_{iFD} = \frac{1}{\sigma^{-1} e^{\epsilon_i/kT} + 1} \longrightarrow N_{FD}(x) = \frac{1}{e^x + 1}$$

$$P(E) = \begin{cases} \frac{1}{e^{(E-\mu)/k_BT}} = Ae^{-\frac{E}{k_BT}} & \text{classical particles Maxwell-Boltzmann statistics} \\ \frac{1}{e^{(E-\mu)/k_BT} - 1} \quad \mu < 0 & \text{Bosons Bose-Einstein statistics} \\ \frac{1}{e^{(E-\mu)/k_BT} + 1} \quad \mu = E_F & \text{Fermions Fermi-Dirac statistics} \end{cases}$$



Homogeneous Bose Gas
Bose-Einstein Condensation

Bose Einstein Distribution:

Note $x = \beta E = E/(kT)$ (that means $dx = dE/(kT)$) and the de Broglie wavelength is $\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mkT}}$.

$$P(E) = \frac{1}{e^{\beta(E-\mu)} - 1} = \frac{1}{z^{-1}e^{\beta E} - 1} \quad (1)$$

where μ is the chemical potential and $z = e^{\beta\mu}$ is the fugacity. μ must be negative so that the distribution is limited.

Consider a fixed number of bosons N

$$N = \int \rho(E)P(E)dE \quad (2)$$

and remember(using $\alpha = 1/z$):

$$\rho(E)P(E)dE = V\frac{4\pi}{h^3}\sqrt{2m^3}\sqrt{E}\frac{1}{z^{-1}e^{E/(kT)} - 1} \quad (3)$$

Bose-Einstein Condensation II

$$N = \int_0^\infty \rho(E)P(E)dE \quad (4)$$

set $x=E/kT$

$$= \frac{V}{\lambda_{dB}^3}\frac{2}{\sqrt{\pi}}\int_0^\infty dx\sqrt{x}\frac{z \cdot e^{-x}}{1 - z \cdot e^{-x}} \quad (5)$$

$$N = \frac{V}{\lambda_{dB}^3} g_{3/2}(z) \quad (6)$$

where $g_{3/2} = \sum_{l=1}^{\infty} = \frac{z^l}{l^{3/2}}$ is the Bose function and $z < 1$.
(Important values are $g_{3/2}(0) = 0$ and $g_{3/2}(1) = 2.612$.)

The problem is that z cannot be greater than one (μ must be negative) leads to a condition for the medium number of particles (integral cannot be greater than 2.612):

$$N \leq 2.612\frac{V}{\lambda_{dB}^3} \quad (7)$$

$$n\lambda_{dB}^3 \leq 2.612 \quad (8)$$

Bose-Einstein Condensation III

In the continuous spectrum we accounted for the ground state with a density of $\rho(0) = 0$.The population $\overline{N_0}$ in the ground state has been neglected in the calculation so far. The only solution is that if $N > N_C$ all other particles go into the ground state. It becomes for temperatures below T_C macroscopic and has to be included by:

$$N = \overline{N_0} + \frac{V}{\lambda_{dB}^3} g_{3/2}(z) \tag{9}$$

$\rightarrow$ phase transition. We can find that the critical temperature at which the phase transition occurs is as:

$$kT_C = \frac{2\pi\hbar^2}{m}\left(\frac{n}{2.612}\right)^{2/3} \tag{11}$$

and the population in the ground state as

$$N_0(T) = N\left(1 - \left(\frac{T}{T_C}\right)^{3/2}\right) \tag{12}$$

Bose-Einstein Condensation IV

Bose Einstein Distribution

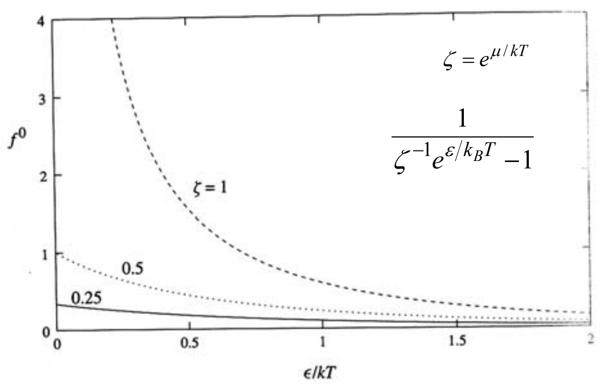


Fig. 2.1. The Bose distribution function f^0 as a function of energy for different values of the fugacity ζ. The value $\zeta = 1$ corresponds to temperatures below the transition temperature, while $\zeta = 0.5$ and $\zeta = 0.25$ correspond to $\mu = -0.69kT$ and $\mu = -1.39kT$, respectively.

Number of particles in the condensate

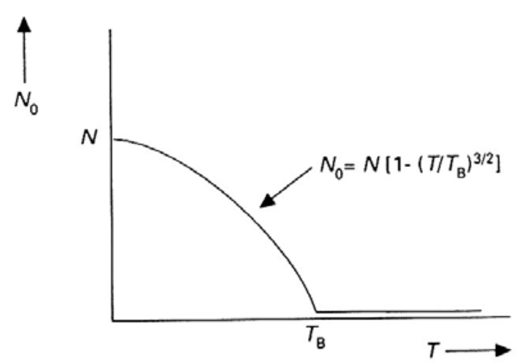


Abb. 8.1: Die Abhängigkeit von N_0, der Anzahl der Bosonen im Grundzustand eines idealen Bose-Einstein-Gases, als Funktion der Temperatur.

Bose Distribution
other examples

Planck radiation law

Behandlung relativistisch (s. 3.2) $\Rightarrow E = h\nu$

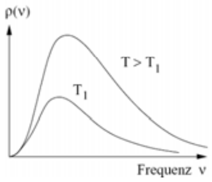
2 Polarisationszustände des Photons $\Rightarrow g_s = 2$

Mit $g(E)dE = g(h\nu)d(h\nu)$ $\quad g(\nu)d\nu = V\frac{8\pi}{h^3}\frac{(h\nu)^2}{c^3}d(h\nu) = \frac{8\pi V}{c^3}\nu^2 d\nu$

Totale Photonenzahl nicht fest, sondern massiv temperaturabhängig
$\Rightarrow \alpha = 0$, $e^\alpha = 1$ (α nicht durch N_0 fixierbar)

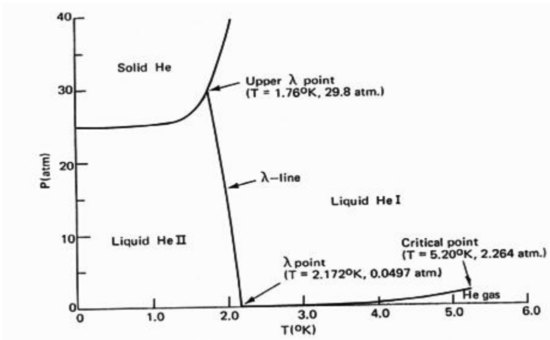
Anzahldichte der Photonen $\quad \frac{dN}{d\nu} = \frac{8\pi V}{c^3}\nu^2 \frac{1}{e^{h\nu/kT}-1}$

Energiedichte des Photonen„gases" $\quad \rho(\nu) = \frac{1}{V}\frac{dN}{d\nu}h\nu$



$\Rightarrow \quad \boxed{\rho(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}d\nu}$ *Planck*

Superfluid He



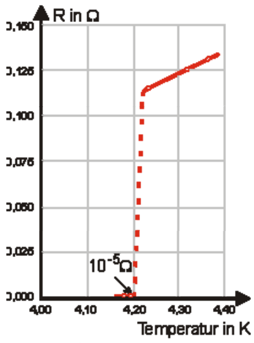
The phase diagram of He[4].

Paired Fermions

2 Fermions
form a Pair
(=Boson)

BCS - pairing

Superconductivity



Superfluid He-3

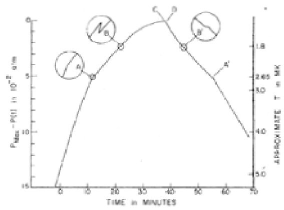
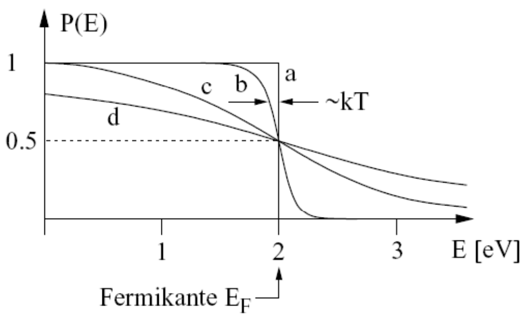


FIG. 2. Time evolution of the pressure in the Pomeranchuk cell during compression and subsequent decompression.

Fermi-Distribution

Annahme: E_F fest (s.u.); numerisches Beispiel für $E_F = 2$ eV, $T_F = E_F/k = 23\,000$ K



	T (K)	kT (eV)
a	0	0
b	1000	0.085
c	5000	0.43
d	20000	1.7

Diskussion der Kurven:

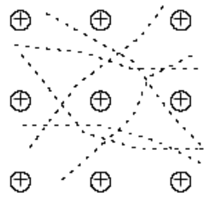
a $\quad T = 0$: alle Phasenraumzellen der Zustände E_i bis zur Grenze E_F mit je **1** Fermion besetzt, also *voll*; $P(E) = 1$. Für $E_i > E_F$ alle Zellen leer, d.h. $P(E) = 0$.

b, c $\quad T > 0$, aber $kT \ll E_F$: Abrundung, Verschmierung der Fermikante

d $\quad T \gg 0$, $kT \approx E_F$: selbst für die niedrigsten Zustände E_i gibt es unbesetzte Zellen
$\Rightarrow$ irgendwann Grenzfall Boltzmann

Allgemeine Regel: Im Bereich $E \gg E_F$ Boltzmann-„Schwanz": dort $e^{-E/kT}$ immer ausreichend!

Fermi-Distribution
Electrons in a solid



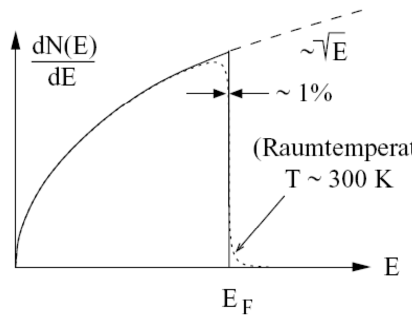
Leitungselektronen quasi-frei, ca. 1–2/Atom in Cu, Ag, …
Räumliche Elektronen-„Gas"-Dichte: $N/V = 2.5 \cdot 10^{22}\ \mathrm{e}^-/\mathrm{cm}^3$
(1 pro Zelle von 3.5 Å)

Vgl. normales Gas $\quad N/V = \approx 3 \cdot 10^{19}\ /\mathrm{cm}^3$
(„verdünnt" gegenüber Festkörper)

$$\frac{dN}{dE} = \frac{V}{h^3} 8\pi\sqrt{2m^3}\sqrt{E}\frac{1}{e^{(E-E_F)/kT}+1}$$

Totale Teilchenzahl N_0 fest: E_F aus $N_0 = \int_0^\infty \frac{dN}{dE}\, dE \Big|_{T=0}$

$$E_F = \frac{h^2}{8m}\left(\frac{3N_0}{\pi V}\right)^{2/3}$$



Typische Zahlen: $E_F = 2 \ldots 7$ eV $\gg kT$!
„Entartungstemperatur":
$T_F = E_F/k = 23\,000 - 82\,000$ K!

⇒ Fermi-Verteilung extrem scharfkantig bei Raumtemperatur

BEC in external Potential
V. Bagnato et al. Phys. Rev. **35**, p4354 (1987)

free space

$$T_c = \frac{h^2}{2\pi k M}\left[\frac{1}{2.612}\frac{N}{V}\right]^{2/3}$$

potential

density of states

$$\rho(\varepsilon) = \frac{2\pi(2M)^{3/2}}{h^3}\int_{V^*(\varepsilon)}\sqrt{\varepsilon - U(\mathbf{r})}\,d^3r$$

$$N = N_0 + \int_0^\infty n_\varepsilon \rho(\varepsilon)\,d\varepsilon$$

total energy and heat capacity

$$E(T) = \int_0^\infty \varepsilon n_\varepsilon \rho(\varepsilon)\,d\varepsilon$$

The heat capacity $C(T) = \partial E(T)/\partial T$

$$C(T) = \frac{1}{kT}\int_0^\infty \frac{\varepsilon\rho(\varepsilon)}{g_\varepsilon}(n_\varepsilon)^2\left[\mu'(T) + \frac{\varepsilon-\mu}{T}\right] \times \exp\left[\frac{\varepsilon-\mu}{kT}\right]d\varepsilon\,,$$

where $\mu'(T) = \partial\mu/\partial T$. $C(T)$ is analogous to C_p in that it includes work done against the potential as the energy of the gas is increased. However, for obvious reasons the volume and pressure are not useful thermodynamic variables.

formulas for power law potentials

power-law potential

$$U(\mathbf{r}) = \varepsilon_1\left|\frac{x}{a}\right|^p + \varepsilon_2\left|\frac{y}{b}\right|^l + \varepsilon_3\left|\frac{z}{c}\right|^q .$$

density of states

$$\rho(\varepsilon) = \left[\frac{2\pi(2M)^{3/2}}{h^3}\right]\frac{abc}{\varepsilon_1^{1/p}\varepsilon_2^{1/l}\varepsilon_3^{1/q}}\varepsilon^\eta F(p,l,q)\,,$$

where $\eta = 1/p + 1/l + 1/q + \frac{1}{2}$ and $F(p,l,q)$ is defined by

$$F(p,l,q) = \left[\int_{-1}^{1}(1-X^p)^{1/2+1/q+1/l}dX\right]\left[\int_{-1}^{1}(1-X^l)^{1/q+1/2}dX\right]\left[\int_{-1}^{1}(1-X^q)^{1/2}dX\right]$$

The critical temperature is

$$T_c = \left[\frac{h^3}{2\pi(2M)^{3/2}}\frac{N}{abc}\frac{\varepsilon_1^{1/p}\varepsilon_2^{1/l}\varepsilon_3^{1/q}}{k^{\eta+1}F(p,l,q)Q(\eta)}\right]^{1/(\eta+1)}$$

where $Q(\eta) = \int_0^\infty \{\theta^\eta/[\exp(\theta)-1]\}d\theta$.

The ground-state population fraction for $T < T_c$ is

$$\frac{N_0}{N} = 1 - (T/T_c)^{\eta+1}$$

BEC in external Potential II

V. Bagnato et al. Phys. Rev. **35**, p4354 (1987)

TABLE I. Critical temperature, ground-state population, heat capacity, and discontinuity in $C(T)$ for several cases of three-dimensional 3(D) confinement. (V represents volume and S, area). In the first two cases where the potential is one dimensional, rigid walls are assumed in the other direction. For the harmonic oscillator, the result agrees with previous calculation (Ref. 7).

Potential	T_c	$N_0/N\ (T<T_c)$	$C(T_c^-)/Nk$	$\Delta C(T_c)/Nk$
$U(z)=\begin{cases}\varepsilon_3(z/a), & z>0\\ \infty, & z<0\end{cases}$	$\left[\frac{h^3N}{1.4Sk^{5/2}(2\pi M)^{3/2}}\right]^{2/5}\left[\frac{\varepsilon_3}{a}\right]^{2/5}$	$1-\left[\frac{T}{T_c}\right]^{5/2}$	6.88	3.35
$U(z)=\varepsilon_3(z/a)^2$	$\left[\frac{3h^3N}{\sqrt{2}Sk^2\pi^4M^{3/2}}\right]^{1/2}\left[\frac{\varepsilon_3}{a^2}\right]^{1/4}$	$1-\left[\frac{T}{T_c}\right]^{2}$	4.38	0
3D box	$\left[\frac{h^3N}{2.612k^{3/2}(2M\pi)^{3/2}V}\right]^{2/3}$	$1-\left[\frac{T}{T_c}\right]^{3/2}$	1.92	0
$U(r)=\varepsilon_1(r/a)^2$	$\left[\frac{Nh^3}{1.202\pi^3k^3(2M)^{3/2}}\right]^{1/3}\left[\frac{\varepsilon_1}{a^2}\right]^{1/2}$	$1-\left[\frac{T}{T_c}\right]^{3}$	10.82	6.57
$U(z,\rho)=\varepsilon_1(z/a)^2+\varepsilon_2(\rho/b)^2$	$\left[\frac{Nh^3}{1.202\pi^3k^3(2M)^{3/2}}\right]^{1/3}\left[\frac{\varepsilon_1}{a^2}\right]^{1/6}\left[\frac{\varepsilon_2}{b^2}\right]^{1/3}$	$1-\left[\frac{T}{T_c}\right]^{3}$	10.82	6.57

General Scaling

Density of states in a general setting: $\rho(E)=C_\alpha E^{\alpha-1}$

using $x=E/kT_c$

$$N_{ex}=C_\alpha (kT_c)^\alpha \int_0^\infty dx \frac{x^{\alpha-1}}{e^x-1}$$

$$=C_\alpha\,\Gamma(\alpha)\,\zeta(\alpha)\,(kT_c)^\alpha$$

Rieman's zeta-function $\zeta(\alpha)=\sum_{n=1}^{\infty} n^{-\alpha}$

and for the critical temperature

$$kT_c=\frac{N^{1/\alpha}}{\left[C_\alpha\,\Gamma(\alpha)\,\zeta(\alpha)\right]^{1/\alpha}}$$

and

$$N_{ex}=N\left(\frac{T}{T_c}\right)^\alpha$$

$$N_0=N\left[1-\left(\frac{T}{T_c}\right)^\alpha\right]$$

α	$\Gamma(\alpha)$	$\zeta(\alpha)$	system
1	1	infinite	2-dim
3/2	$\pi^{1/2}/2=0.886$	2.612	box
2	1	$\pi^2/6=1.645$	2d harm. osz.
5/2	$3\pi^{1/2}/4=1.329$	1.341	
3	2	1.202	3d harm. osz.
7/2	$15\pi^{1/2}/8=3.323$	1.127	
4	6	$\pi^4/90=1.082$	

Low Dimensions

$$V_{\text{ext}}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) = \tfrac{1}{2} m\omega_\perp^2 r_\perp^2 + \tfrac{1}{2} m\omega_z^2 z^2 = \tfrac{1}{2} m\omega_\perp^2 (r_\perp^2 + \lambda^2 z^2)$$

Because of the infrared divergence of the integral

$$N = \int_0^\infty \rho(\varepsilon)\frac{1}{\exp(\varepsilon-\mu)/k_BT - 1}\,d\varepsilon$$

there is no BEC for 2D or 1D in free space

the situation changes dramatically dir trapped quantum gas

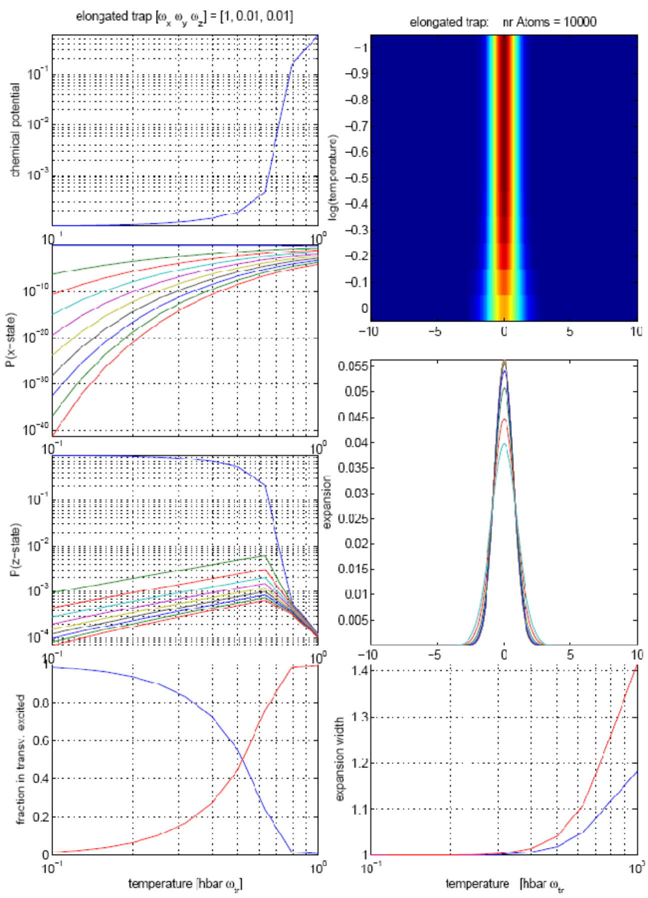
- DOS is different
- use a finite sum

$$N = \sum_{n_x n_y n_z} \frac{1}{\exp[(\epsilon_{n_x n_y n_z} - \mu)/T] - 1}$$

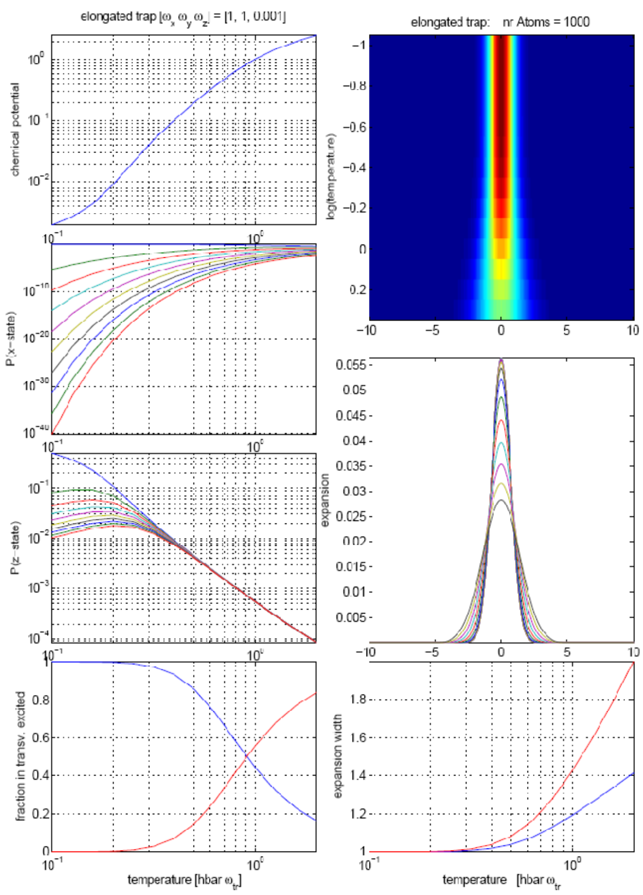
- with a finite ground state energy

	3D	2D $k_BT \ll \hbar\omega_z$ $k_BT > \hbar\omega_\perp$	1D $k_BT \ll \hbar\omega_\perp$ $k_BT > \hbar\omega_z$
DOS free space	$\varepsilon^{1/2}$	constant	$\varepsilon^{-1/2}$
T_c	$kT_C = \frac{2\pi\hbar^2}{m}\left(\frac{n}{2.612}\right)^{3/2}$	$k_BT_c \cong \frac{\hbar^2}{mL^2}\frac{N}{\ln(N)}$	-
N/N_0	$N_0 = N\left(1 - \frac{T}{T_0}\right)^{3/2}$		
DOS harmonic trap	ε^2	ε	const
T_c	$T_c^{3D} = \frac{\hbar}{[\zeta(3)]^{1/3}}\omega_{ho}N^{1/3}$	$T_c^{2D} = \frac{\hbar\sqrt{6}}{\pi}\omega_{ho}N^{1/2}$	$k_BT_c \cong \hbar\omega_z \frac{N}{\ln(N)}$
N/N_0	$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c^{3D}}\right)^3$	$\frac{N_0}{N} \approx 1 - \left(\frac{T}{T_c^{2D}}\right)^2$	$\frac{N}{N_0} \sim 1 - \frac{T}{T_c}$

2D-Trap



1D-Trap



Fermions in a Trap

Fermi-Dirac Distribution

$$f(\epsilon) = \frac{1}{\exp[(\epsilon-\mu)/k_B T]+1}$$

with $N = \int g(\epsilon) f(\epsilon) d\epsilon$

and the density of states of a harmonic oscillators

$$g(\epsilon) = \epsilon^2 / 2(\hbar\omega_{ho})^3$$

one obtains the Fermi energy $E_F = \hbar\omega_{ho}(6N)^{1/3}$

and Fermi temperature $T_F = E_F / k_B$

At T=0 and equilibrium we require

$$\frac{\hbar^2 k_F^2(\mathbf{r})}{2m} + V(\mathbf{r}) = E_F$$

and using the relation between Fermi momentum and density $\frac{4}{3}\pi k_F^3(\mathbf{r}) = (2\pi)^3 n(\mathbf{r})$

we find the density profile

$$n(\mathbf{r}) = \frac{1}{6\pi^2}\left[\frac{2m}{\hbar^2}(E_F - V(\mathbf{r}))\right]^{3/2}$$

anisotropic harmonic oscillator

$$E_F = \frac{1}{2}m\omega_x^2 R_x^2 = \frac{1}{2}m\omega_y^2 R_y^2 = \frac{1}{2}m\omega_z^2 R_z^2$$

with $\omega_{ho} = (\omega_x\omega_y\omega_z)^{1/3}$

we find $R_i = a_{ho}(48N)^{1/6}\frac{\omega_{ho}}{\omega_i}$.

for cigar shaped traps:

trapping frequencies $\omega_\perp$ and $\omega_\parallel$

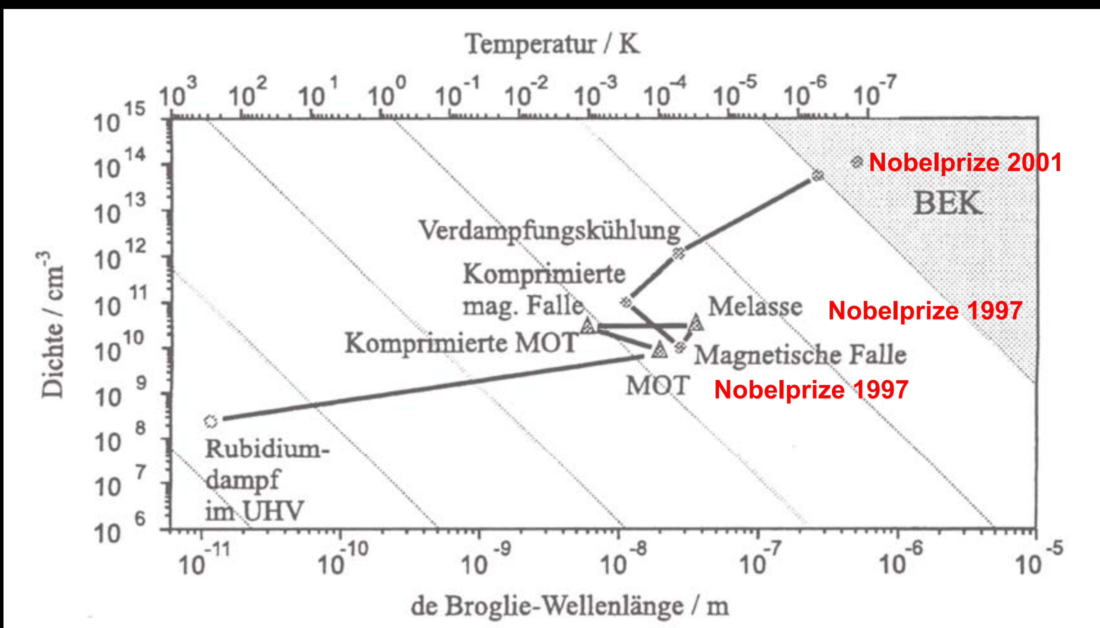
$$n(\mathbf{r}) = \frac{8}{\pi^2}\frac{N}{R^2 Z}\left(1 - \frac{r^2}{R^2} - \frac{z^2}{Z^2}\right)^{3/2}$$

$$k_F = \sqrt{2mE_F/\hbar^2} = (48N)^{1/6}/a_{ho}$$

width of the momentum distribution

$$n(\mathbf{k}) = \frac{8}{\pi^2}\frac{N}{k_F^3}\left(1 - \frac{k^2}{k_F^2}\right)^{3/2}$$

Making a BEC



BEC what we need

- **extremely cold atoms**
  - Ways to cool and accumulate large number of atoms
  - Laser cooling
  - Laser trapping (MOT)
  - Evaporative cooling
- **conservative trap to hold the atoms**
  - Ways to hold large number of ultra cold atoms without heating
  - Magnetic trap
  - Optical trap
- **good collision properties**
  - High collision rate to achieve thermalization (good collisions)
  - Low inelastic rates (bad collisions)

Cold Atoms for BEC
basics

Laser Cooling
Neutral atoms can be cooled by interacting with monochromatic light (~thermal equilibrium with the light)

•Temperature	1mK ⇨ 1μK
•Velocity	0.5m/s ⇨1mm/s
•deBroglie wavelength	10nm ⇨500nm
•Typical samples	10^8 atoms @ 10^{11} atoms/cm^3

Magnetic Trapping
Neutral atoms can be magnetically trapped $U=-\mu B$
1Gauss ~ 67 μK for a magnetic moment $\mu = \mu_B$

Evaporative cooling to BEC
Cooling in a magnetic trap by removing the hottest atoms and thermal equilibration (evaporative cooling)

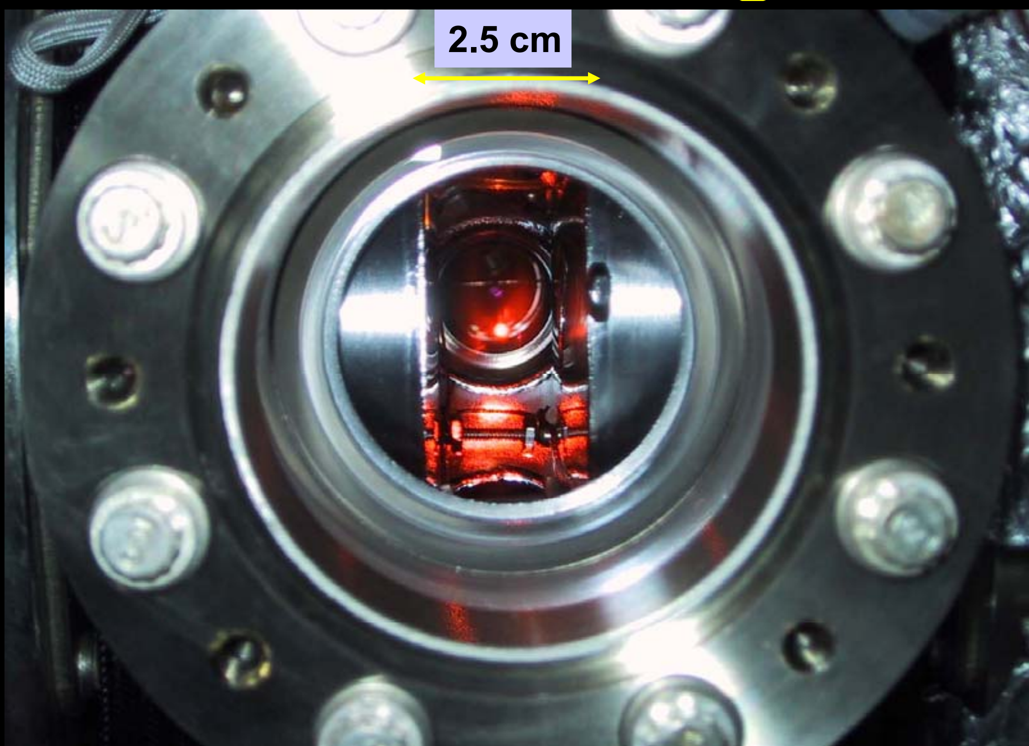
•Typical samples	>10^5 atoms @ 10^{14} atoms/cm^3
•Temperature	<1μK
•deBroglie wavelength	>1μm

Cold Atoms
basic relations

	E [eV]	v [cm s-1]	λ [Å]	h [cm]
E [eV]		$5.182\ 10^{-13}\ A\ v^2$	$\frac{0.0825}{A\ \lambda^2}$	$1.017\ 10^{-9}\ A\ h$
v [cm s-1]	$1.389\ 10^6 \sqrt{\frac{E}{A}}$		$\frac{3.990\ 10^5}{A\ \lambda}$	$44.29\ \sqrt{h}$
λ [Å]	$\frac{0.2873}{\sqrt{E\ A}}$	$\frac{3.990\ 10^5}{A\ v}$		$\frac{9008.6}{A\ \sqrt{h}}$
h [cm]	$9.836\ 10^8\ \frac{E}{A}$	$5.097\ 10^{-4}\ v^2$	$\frac{8.115\ 10^7}{A^2\ \lambda^2}$	

$$t = 0.04515\ \sqrt{h}$$

Laser cooling



Laser cooling requires low density to avoid light absorption

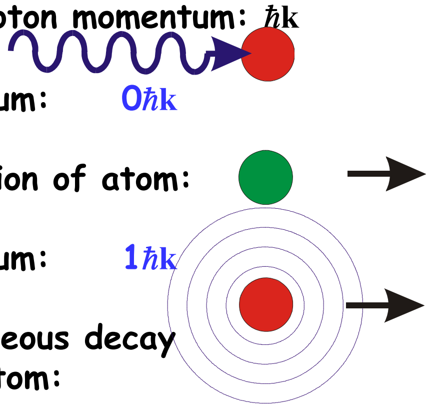
Recomended Literature
Laser Cooling

- *The Quantum Theory of Light;*
  R. Laudon: Oxford Science Publications
- *Laser Cooling and Trapping*
  H. Metcalf, P. van der Straaten (Springer)
- *Nobel prize lectures 1997:*
  - *The manipulation of neutral particles*
    S. Chu; Rev. Mod. Phys. 71 685 (1998)
  - *Manipulating atoms with photons*
    C. Cohen-Tannoudji: Rev. Mod. Phys. 71 707 (1998)
  - *Laser cooling and trapping of neutral atoms*
    W. Phillips: Rev. Mod. Phys. 71 721 (1998)

Cold Atoms
mechanical effects of light

Scattering of a photon by an atom

photon momentum: $\hbar k$



atom momentum: $0\hbar k$

after excitation of atom:

atom momentum: $1\hbar k$

after spontaneous decay of atom:

momentum: $1\hbar k$

mean atom

Mean force on atom:

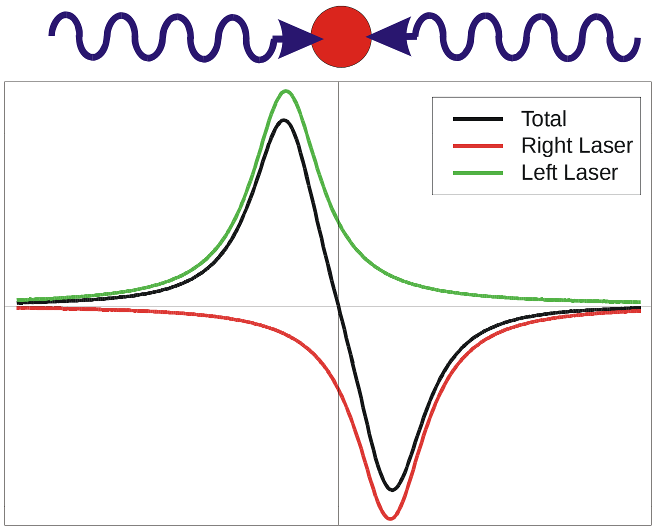
$$F = \frac{dp}{dt} \approx \frac{\Delta p}{\Delta t} = \hbar k \Gamma \rho_{22} = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left(2\frac{\omega_L - \omega_a - \vec{k}\vec{v}}{\Gamma}\right)^2}$$

typical forces on the atom can lead to accelerations of 10^4-10^6 m/s^2

Cold Atoms

laser cooling

Atom in counter propagating laser field: optical molasses



Close to velocity zero: force is linear in velocity

$$F=-\alpha v$$

For a detuning

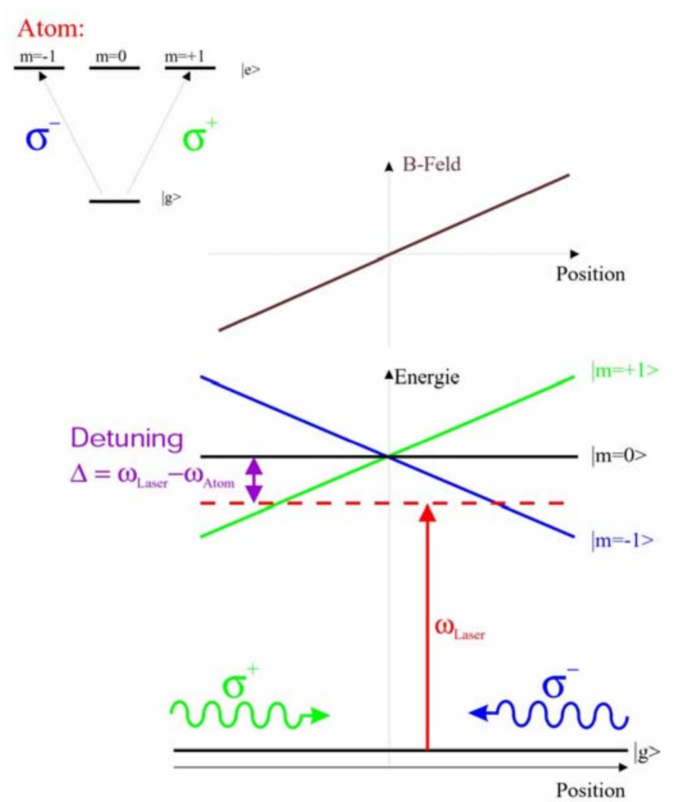
$$\delta=\omega_{laser}-\omega_{atom}<0$$

(red from resonance)
$\alpha>0$ and the force is a damping force

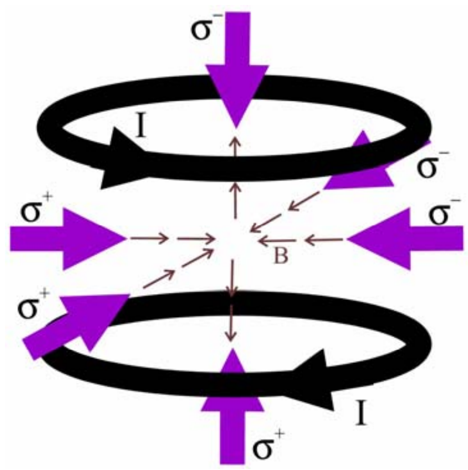
Heating due to randomness of the photon scattering
typical temperature: $k_BT=\hbar\Gamma/2$ (Doppler limit)
140 μK for Γ=5 MHz

Magneto Optic Trap

E. Raab et al. PRL **59** p2631 (1987)



3d magnetic field realization: Quadrupole



Atoms are pushed to the point with B=(

Typical parameters:
- Density: $>10^{11}$ atoms/cm^3
- Up to $>10^{10}$ atoms

How to collect cold atoms

Problem:
capture range of light forces is small (10m/s) compared with the velocity of thermal atoms (500m/s)

Slow atoms from thermal velocity

- Zeeman slower
  (tune the atom transitions with external magnetic fields)

Collect the slow atoms out of the low energy tail of the Maxwell distribution

- Vapour cell MOT
  the 4π solid angle captured by the MOT compensates for the small number of slow atoms. Flux ~ v^3 for $mv^2 << kT$
  Maxwell Distr: $f(v) \sim v^2 \exp(-mv^2/kT)$

Magnetic Trapping

Atoms in Magnetic Field

Breit Rabi Formula for F=I+1/2

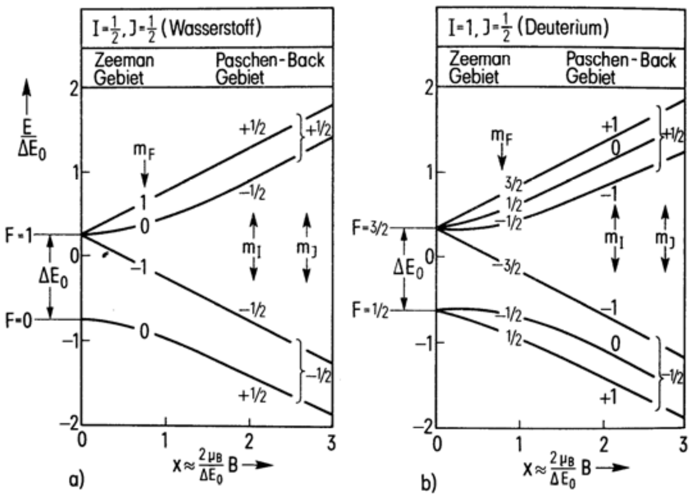


Fig. 102 Feldabhängigkeit der HFS-Aufspaltung nach der Breit-Rabi-Formel für I = 1/2 und I = 1

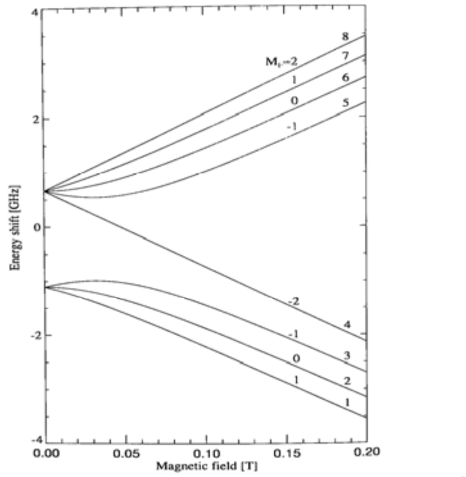


FIGURE 4.2. Energies of the ground hyperfine states of Na, where the states are numbered 1–8 and M_F is the projection of the total angular momentum of the atom on the magnetic field axis.

$$E_B^{HFS}(F=I\pm\tfrac{1}{2},m_F) = -\frac{A}{4} + m_F g_K \mu_K B \pm \frac{\Delta E_0}{2}\sqrt{1+\frac{4m_F}{2I+1}x+x^2}$$

$$x = \frac{g_J\mu_B - g_K\mu_K}{\Delta E_0} \approx \frac{2\mu_B}{\Delta E_0} \qquad \Delta E_0 = A(I+\tfrac{1}{2})$$

Magnetic Trapping

Trapping potential: $U_{mag} = -\vec{\mu}\cdot\vec{B}$

for $\mu = \mu_B$: 1 Gauss → $U_{mag} = 67\,\mu K = 5.78\times10^{-9}$ eV

Magnetic states:

- $U_{mag} < 0$ high field seeking (attracted to maximum)
- $U_{mag} > 0$ low field seeking (attracted to minimum)

Earnshaw Theorem:

No maximum of a static field (combination of fields) in a source free region

Magnetic traps are low field seeker traps,

Atoms trapped in minimum of field but not in the ground state of potential (this would be a high field seeking state)

Avoid Zeros in the field (Majorana transitions)

Quadrupole trap has a zero in the field at the centre ! Even at non zero weak field, there are Landau-Zener transitions possible.

Rate for a harmonic minimum: $\gamma = \frac{\pi\omega}{2\sqrt{e}}\, e^{\frac{-\mu_{\|} B_{ip}}{\hbar\omega}}$

B_{ip} … field at minimum

ω … trap frequency

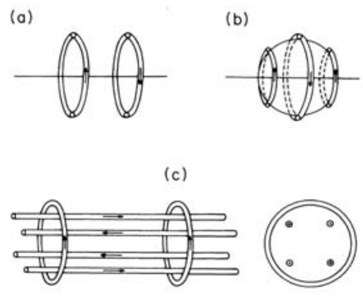
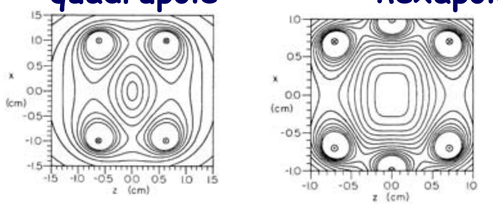


FIG. 1. Three magnetostatic trap configurations discussed in this work. (a) The magnetic quadrupole trap, consisting of two coils with opposing currents. (b) The "spherical hexapole" trap, with three wires on the surface of a sphere. With equal currents and the outer coils at 45°, **B**=0 at the origin. (c) The Ioffe trap, which has a bias field and axial confinement from a two-coil "bottle field" and transverse confinement from a four-wire quadrupole focusing field. Both side and end views are shown for the Ioffe trap.

quadrupole **hexapole**



Time Orbiting Potential trap

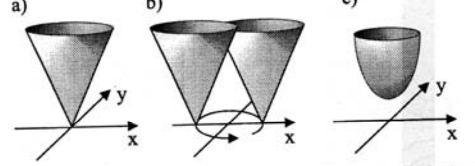


Abbildung 3.4: *Funktionsprinzip einer TOP-Falle. Das lineare Potential a) wird durch das homogene Feld in Rotation versetzt b). Zeitgemittelt ergibt sich in erster Näherung das in c) gezeigte harmonische Potential.*

Configurations with non-Zero Field Minimum

Ioffe Pritchard Trap

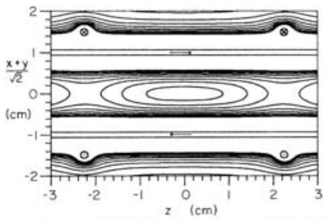


FIG. 7. Contours of |**B**| for the Ioffe trap of Fig. 1(c) in the plane of the straight wires. The four straight wires lie on a circle of radius 1 cm, the coils of radius 1.5 cm are spaced by 4.5 cm, and all currents are 100 A. The minimum field at the origin is 14.3 G. Contours are shown at 10 G intervals up to 100 G.

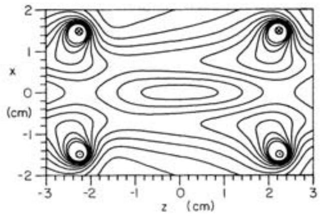


FIG. 8. Contours of |**B**| at 10 G intervals to 100 G in a plane midway between the straight wires for the Ioffe trap with parameters as given in Fig. 7. For a plane perpendicular to the one chosen, the contours will be as shown, but reflected in the $z=0$ line.

Baseball Trap

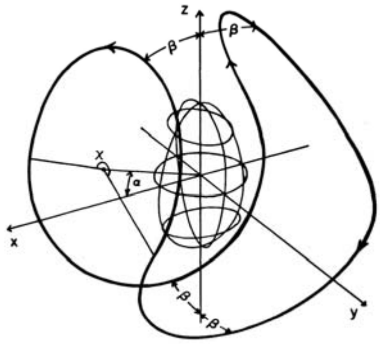


FIG. 9. The baseball coil (heavy line) with an equipotential surface shown schematically at the center. The coil consists of four contiguous planar circular segments, each subtending an angle χ, on axes at angle $\alpha(-\alpha)$ with respect to the $\pm x$ $(\pm y)$ directions. At closest approach, the coil comes within angle β of the z axis. For this figure, $\alpha=20°$ (hence $\beta=21.6°$), and the contour lines represent a 40-G surface for a baseball of radius $R_B=1$ cm, current $I_B=100$ A.

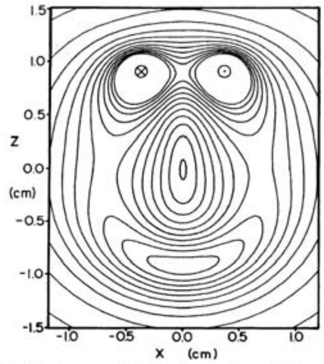


FIG. 10. Contours of |**B**| in the x-z plane of Fig. 9, with $\alpha=20°$ $R_B=1$ cm, and $I_B=100$ A. Contours are drawn at 10-G intervals. At the center, $|\mathbf{B}|_{min}=9.1$ G, while the transverse and axial saddle-point thresholds are 72 and 106 G, respectively.

Other Realizations

Clover Leaf Trap (MIT)

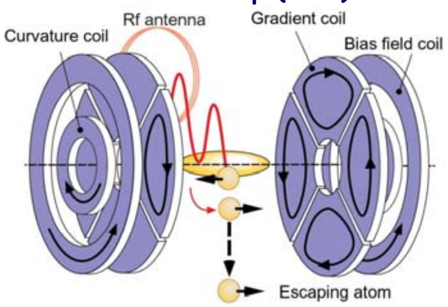
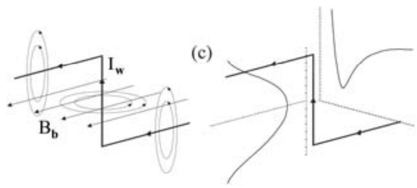


Fig. 4. – In a cloverleaf trap, Ioffe bars are replaced by eight "cloverleaf" coils surrounding the pinch coils, providing 360 degree optical access. Evaporation is done by selectively spin-flipping atoms into untrapped states with rf radiation.

Z-Wire Trap (Innsbruck/HD)



Trapping field is created by superposition of the field of a current carrying Z-shaped Wire and a homogeneous bias field

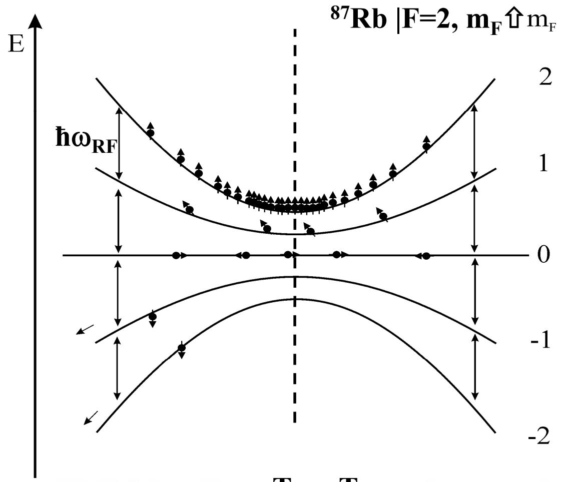
Evaporative cooling

Evaporative Cooling

basic principles

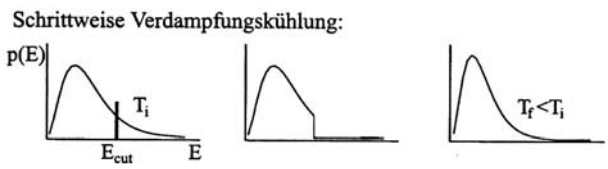
- **Energy / position-selective removal of hot atoms by RF radiation, inducing spin-flips to untrapped states;**
  $E(r) = g_F\, m_F\, \mu_B\, B(r)$
  $\Delta E(m_F)(r) = \hbar\omega_{RF}$

- **Rethermalisation to Maxwell-Boltzmann distribution leads to lower mean temperature**



Evaporative Cooling

H.F. Hess Phys.Rev.B 34 p R3476 (1986)
K.B. Davis, M-O. Mewes, W. Ketterle Appl.Phys.B 60, p155 (1995)
O.J.Luiten, M. Reynolds, J. Walraven Phys.Rev.A 53 p381 (1996)



General Scaling

Evaporative cooling happens at exponential scale
Time scale is given by the thermalisation (collision) time
(it takes about 5 collision for a truncated distribution to thermalise)
Characteristic quantities are logarithmic derivatives
A key parameter is $\alpha = \frac{d(\ln T)}{d(\ln N)}$ (temperature decrease per particle loss)
Evaporation is controlled by a potential depth ηkT

The density of states ($\rho(\varepsilon)$) of trap pot. determines scaling.
For power law potentials: $\rho(\varepsilon)=A_{pl}\,\varepsilon^{1/2+\delta}$
square well: δ=0, harmonic: δ=3/2 , linear: δ=3
Ioffe-Pritchard trap: $\rho(\varepsilon)=A_{IP}\,(\varepsilon^3 + 2U_{IP}\,\varepsilon^2)$

Exponential scaling for a d-dimensional potential $U(r) \sim r^{d/\delta}$

Number of atoms N	1	α with $\frac{d(\ln T)}{d(\ln N)} = \frac{\eta+\kappa}{\delta+\frac{3}{2}} - 1$
Temperature T	α	
Volume V	$\alpha\delta$	
Density n	$1-\alpha\delta$	average energy in potential
Phase space density D	$1-\alpha(\delta+3/2)$	$\langle\varepsilon\rangle = (\delta+3/2)\,kT$
Elastic coll. Rate $n\sigma v$	$1-\alpha(\delta-1/2)$	

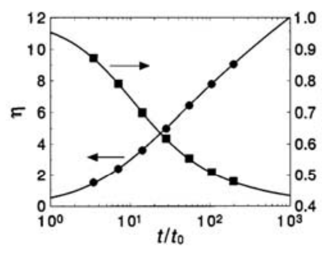


FIG. 5. Truncation parameter η (circles) and fraction of atoms remaining in trap N/N_0 (squares) as a function of reduced time t/t_0 after initiating evaporation from infinite temperature. Curves are obtained by integration of the differential equations resulting from the truncated Boltzmann approximation, symbols by fitting to the distribution obtained by numerical solution of the kinetic equation.

Run-away evaporation:

Achieve faster and faster thermalsation with cooling
Collision rate has to grow: $\alpha(\delta-1/2)>1$
Collisions:
good collisions: elastic collisions
bad collisions: inelastic coll., trap loss coll., etc … limit the trap lifetime
How many collisions per tapping time for run away evap.?
Linear trap: > 25 collisions per lifetime
Harmonic trap >150 collisions per lifetime

RF induced evaporation

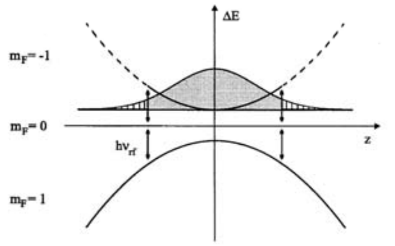


Abbildung 3.11: *Radiofrequenz-induzierte Verdampfungskühlung in einem magnetischen Potential.*

BEC in a Dilute System

example Na

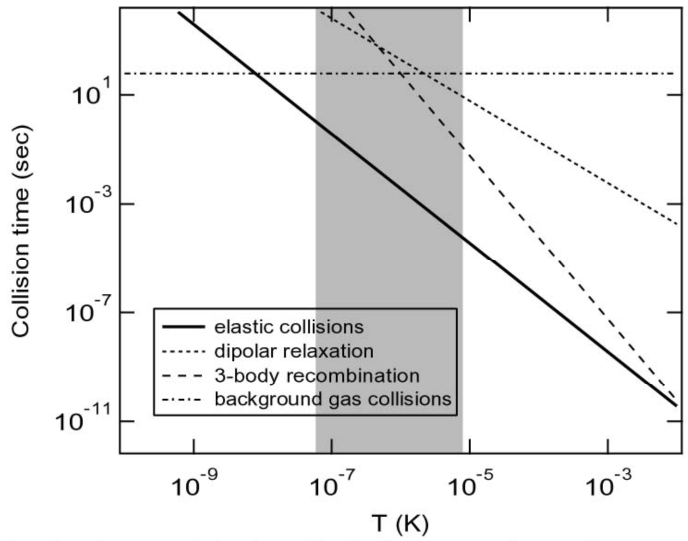
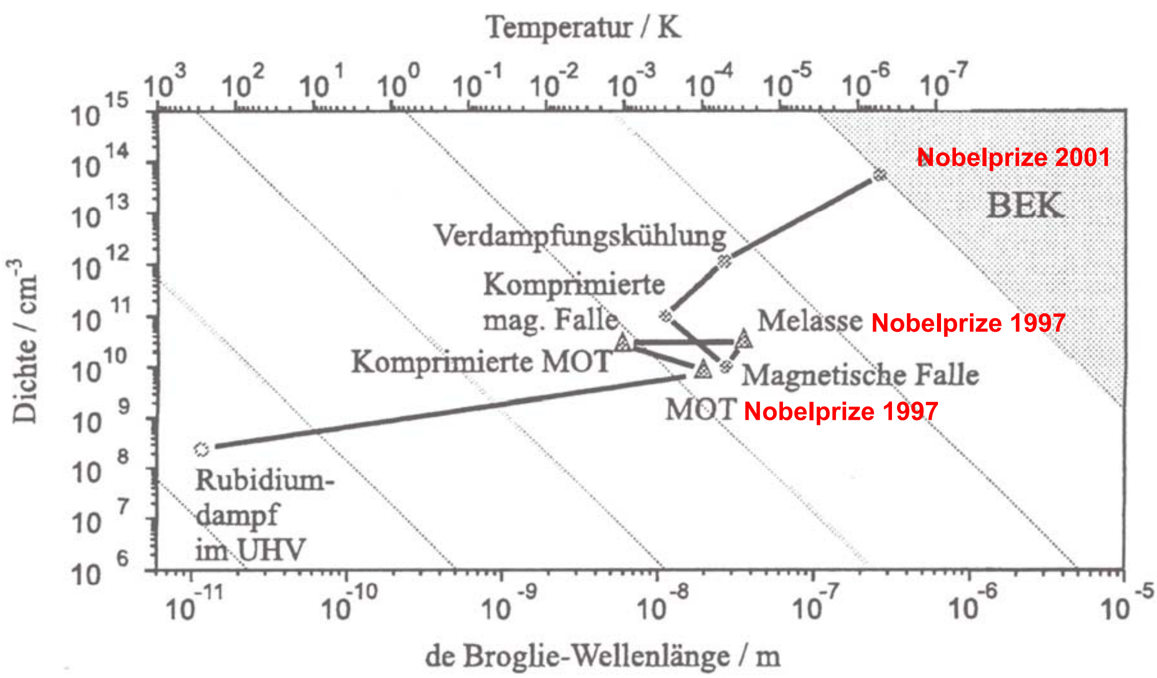


Fig. 6. – Mean collision time for several elastic and inelastic processes in a sodium gas as a function of temperature at the critical density for Bose-Einstein condensation. The "BEC window," where the lifetime of the sample exceeds 0.1 seconds and the rate of elastic collisions is faster than 1 Hz is shaded. This figure uses a scattering length of $50\,a_0$ and rate coefficients for two- and three-body inelastic collisions of $10^{-16}\text{cm}^3\text{s}^{-1}$ and $6\cdot10^{-30}\text{cm}^6\text{s}^{-1}$ respectively.

Roadmap to BEC with an atomic gas

Laser cooling, magnetic trapping, evaporative cooling



Observing the BEC

Methods of imaging:

3 processes: absorption, emission, shifting the phase
3 methods: absorptive, fluorescence, dispersive imagi
Description: complex index of refraction

$$n_{ref} = 1 + \frac{\sigma_0 n \lambda}{4\pi}\left[\frac{i}{1+\delta^2} - \frac{\delta}{1+\delta^2}\right] \quad \text{with } \delta = \frac{\omega-\omega_0}{\Gamma/2}$$

Transmission T and phase shift Φ

$$T = e^{-\tilde{D}/2} = \exp\left(-\frac{1}{2}\frac{\tilde{n}\sigma_0}{1+\delta^2}\right) \quad \text{with } \tilde{D} = \frac{\tilde{n}\sigma_0}{1+\delta^2}$$

$$\Phi = -\delta\frac{\tilde{D}}{2} = -\frac{\delta}{2}\frac{\tilde{n}\sigma_0}{1+\delta^2}$$

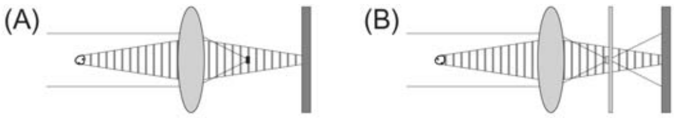
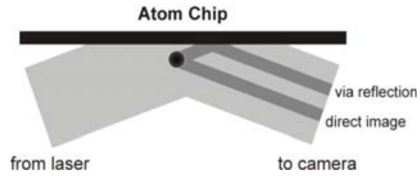
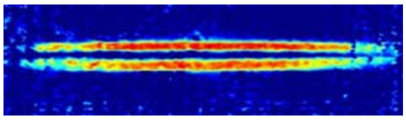


Fig. 7. – Dark-ground (A) and phase-contrast (B) imaging set-up. Probe light from the left is dispersively scattered by the atoms. In the Fourier plane of the lens, the unscattered light is filtered. In dark-ground imaging (A), the unscattered light is blocked, forming a dark-ground image on the camera. In phase-contrast imaging (B), the unscattered light is shifted by a phase plate (consisting of an optical flat with a λ/4 bump or dimple at the center), causing it to interfere with the scattered light in the image plane.

Imaging dense clouds (D_0>100):

Optimal absorption imaging is at optical density of 1
Need large detuning but there is *refraction* at |δ|>0
For diffraction limited imaging we need a phase shift $\Phi < \pi/2$
For optical density 1 we need $|\delta|=(D_0)^{1/2}$
At this detuning the phase shift $\Phi \sim 0.5\,(D_0)^{1/2}$ much too large
For $\Phi < \pi/2$ we need $|\delta|=D_0/\pi$
Need phase contrast imaging

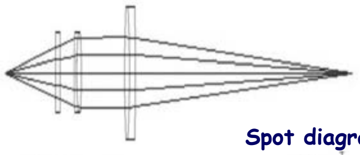




Non destructive imaging:

Example of an image: 30x30 pixel with 100 photons each (10^5 photons)
This can be *'non perturbative'* for large condensates.
Important figure of merit: ratio signal / heating
- Absorption imaging: each photon gives one recoil energy
- Dispersive imaging: there are more forward scattered photons than absorbed photons
  - for large detuning the gain is $D_0/4$!
  - elastic scattered photons contribute not to heating if imaging is done in the trap and light pulse is longer that $1/\nu_{trap}$
  - one can make many pictures of the condensate
- For low density clouds there is *no* advantage of dispersive imaging

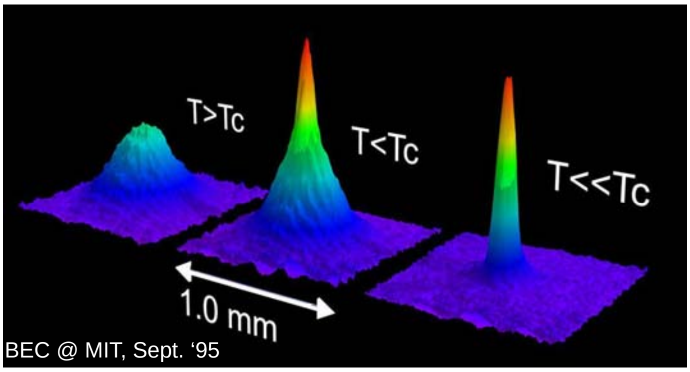
Lens setup (3μm resolution)

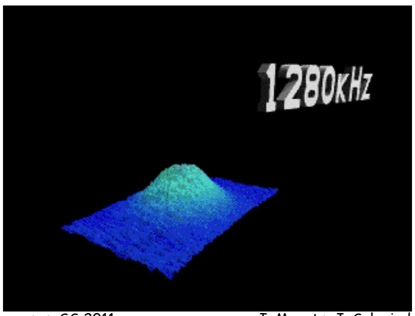


Spot diagram

Observing BEC
phase transition, expansion

phase transition





Expansion

man "sieht" den Grndzustand und in der Expansion erkennt man die Unschärferelation
Kurze Zeiten: sieht δx
Lange Zeiten: sieht δp
kleines δx -> großes δp (schnelle Expansion)
großes δx -> kleines δp (langsame Expansion)