

# 5. Trapping with static fields

## 5.1 Trapping with static fields

No trapping theorems

Magnetic trapping

Micro traps

Ion traps

## 5.2 Evaporative cooling to BEC

## 5.3 RF induced adiabatic potentials

# Atoms in Magnetic Field

Breit Rabi Formula for  $F=I+1/2$

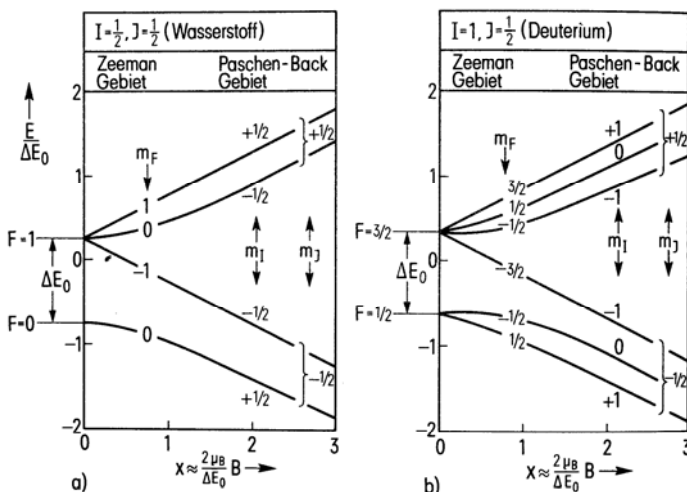


Fig. 102 Feldabhängigkeit der HFS-Aufspaltung nach der Breit-Rabi-Formel für  $I = 1/2$  und  $I = 1$

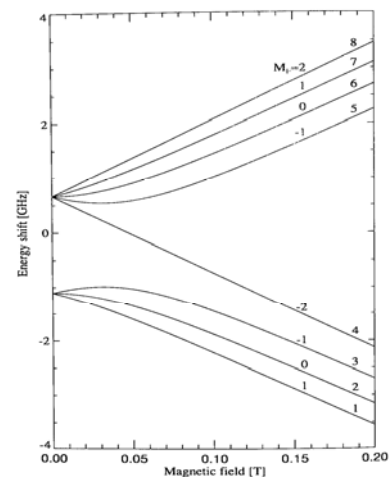


FIGURE 4.2. Energies of the ground hyperfine states of Na, where the states are numbered 1-8 and  $M_F$  is the projection of the total angular momentum of the atom on the magnetic field axis.

$$E_B^{HFS}(F = I \pm \frac{1}{2}, m_F) = -\frac{A}{4} + m_F g_K \mu_K B \pm \frac{\Delta E_0}{2} \sqrt{1 + \frac{4m_F}{2I+1} x + x^2}$$

$$x = \frac{g_J \mu_B - g_K \mu_K}{\Delta E_0} \approx \frac{2\mu_B}{\Delta E_0} B \quad \Delta E_0 = A(I + \frac{1}{2})$$

# No Trapping Theorems

- **Earnshaw's theorem:**  
It is impossible to arrange any set of charges to generate point of stable equilibrium in a charge free region.
- **Optical Earnshaw's theorem:**  
If the light scattering force is proportional to the local Poynting vector, it is impossible to construct an optical trap.
- **No field maximum theorem:**  
In regions free of charges and currents  $|E|$  and  $|B|$  cannot have a local maximum.
- **No compensation theorem:**  
No combination of static electric, magnetic and/or gravitational fields can produce a stable atom trap.

## Magnetic Trapping

Trapping potential:  $U_{\text{mag}} = -\vec{\mu} \cdot \vec{B}$

for  $\mu = \mu_B$ :  $1 \text{ Gauss} \rightarrow U_{\text{mag}} = 67 \mu\text{K} = 5.78 \times 10^{-9} \text{ eV}$

### Magnetic states:

$U_{\text{mag}} < 0$  high field seeking (attracted to maximum)  
 $U_{\text{mag}} > 0$  low field seeking (attracted to minimum)

### Earnshaw Theorem:

No maximum of a static field (combination of fields) in a source free region

### Magnetic traps are low field seeker traps,

Atoms trapped in minimum of field  
 but not in the ground state of potential  
 (this would be a high field seeking state)

### Avoid Zeros in the field (Majorana transitions)

Quadrupole trap has a zero in the field at the centre!  
 Even at non zero weak field, there are Landau-Zener transitions possible.

Rate for a harmonic minimum:  $\gamma = \frac{\pi\omega}{2\sqrt{e}} e^{\frac{-\mu_{\parallel} B_{ip}}{\hbar\omega}}$   
 $B_{ip}$  ... field at minimum  
 $\omega$  ... trap frequency

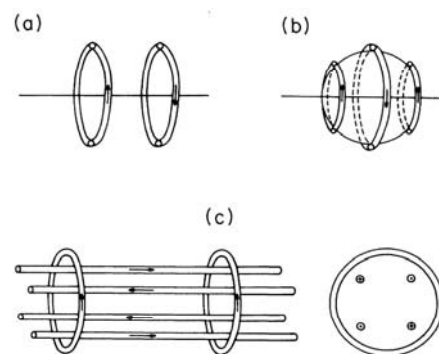
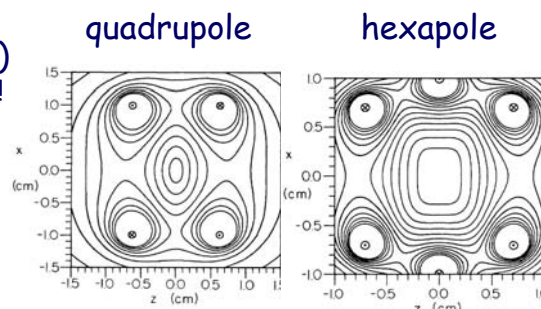


FIG. 1. Three magnetostatic trap configurations discussed in this work. (a) The magnetic quadrupole trap, consisting of two coils with opposing currents. (b) The "spherical hexapole" trap, with three wires on the surface of a sphere. With equal currents and the outer coils at 45°,  $B=0$  at the origin. (c) The Ioffe trap, which has a bias field and axial confinement from a two-coil "bottle field" and transverse confinement from a four-wire quadrupole focusing field. Both side and end views are shown for the Ioffe trap.



# Avoiding the Magnetic Field Zero

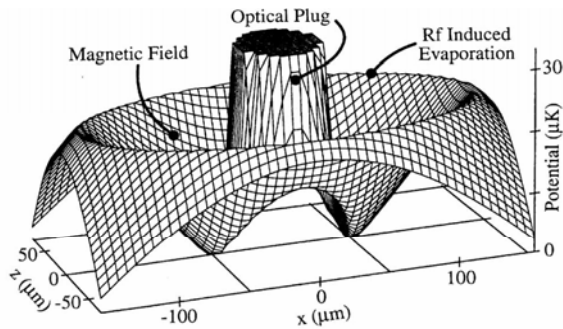


FIG. 1. Adiabatic potential due to the magnetic quadrupole field, the optical plug, and the rf. This cut of the three-dimensional potential is orthogonal to the propagation direction ( $y$ ) of the blue-detuned laser. The symmetry axis of the quadrupole field is the  $z$  axis.

## Time Orbiting Potential trap

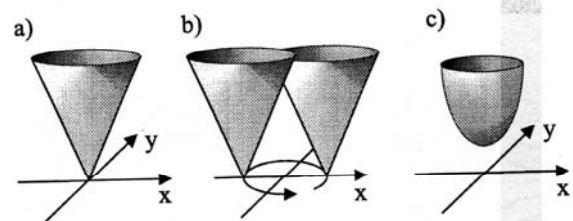


Abbildung 3.4: Funktionsprinzip einer TOP-Falle. Das lineare Potential a) wird durch das homogene Feld in Rotation versetzt b). Zeitgemittelt ergibt sich in erster Näherung das in c) gezeigte harmonische Potential.

# Configurations with non-Zero Field Minimum

## Clover Leaf Trap (MIT)

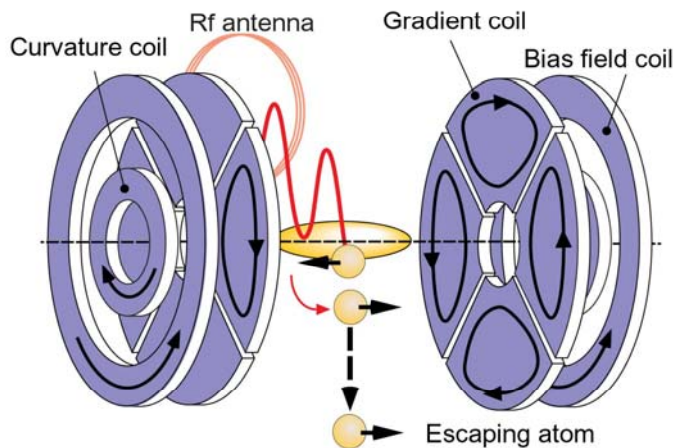


Fig. 4. – In a cloverleaf trap, Ioffe bars are replaced by eight “cloverleaf” coils surrounding the pinch coils, providing 360 degree optical access. Evaporation is done by selectively spin-flipping atoms into untrapped states with rf radiation.

## Ioffe Pritchard Trap

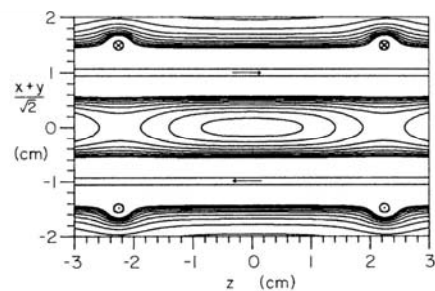


FIG. 7. Contours of  $|\mathbf{B}|$  for the Ioffe trap of Fig. 1(c) in the plane of the straight wires. The four straight wires lie on a circle of radius 1 cm, the coils of radius 1.5 cm are spaced by 4.5 cm, and all currents are 100 A. The minimum field at the origin is 14.3 G. Contours are shown at 10 G intervals up to 100 G.

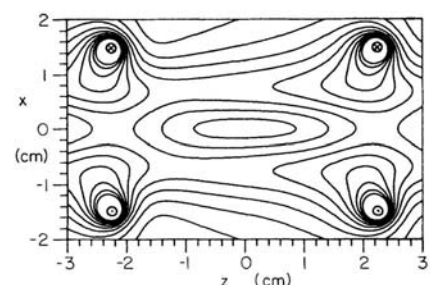


FIG. 8. Contours of  $|\mathbf{B}|$  at 10 G intervals to 100 G in a plane midway between the straight wires for the Ioffe trap with parameters as given in Fig. 7. For a plane perpendicular to the one chosen, the contours will be as shown, but reflected in the  $z=0$  line.

# Tailoring magnetic fields for compression

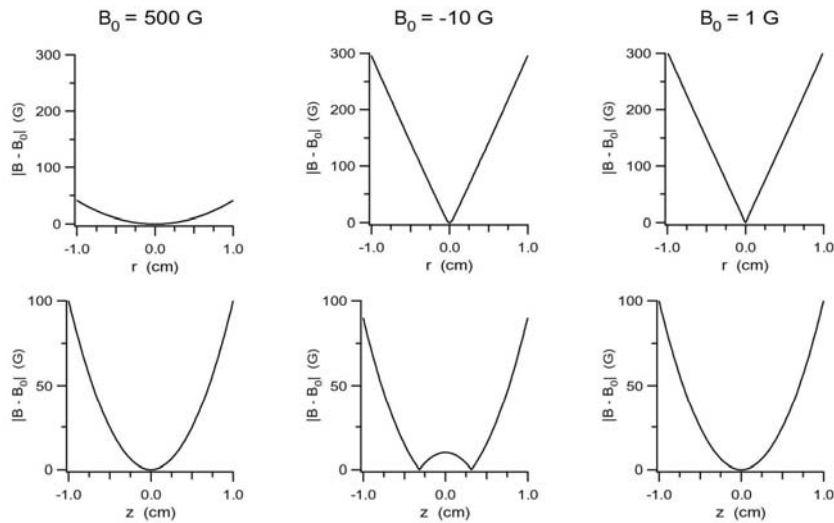


Fig. 3. – Bias field compensation in an Ioffe-Pritchard trap is important for tight radial confinement. The magnetic field in an IP trap characterized by a radial gradient of 300 G/cm and an axial curvature of 200 G/cm<sup>2</sup> is shown for three bias fields  $B_0$ . The upper row displays radial cuts, and the lower row displays axial cuts of the magnetic field profile. In the second column, radial confinement is softened as a result of the large bias field. In the third column, the bias field is tuned correctly, resulting in tight radial confinement and no zero field crossings.

## Baseball Trap (JILA)

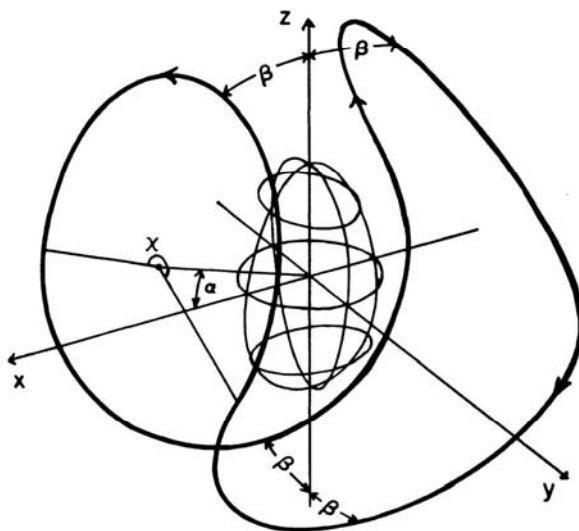


FIG. 9. The baseball coil (heavy line) with an equipotential surface shown schematically at the center. The coil consists of four contiguous planar circular segments, each subtending an angle  $\chi$ , on axes at angle  $\alpha(-\alpha)$  with respect to the  $\pm x (\pm y)$  directions. At closest approach, the coil comes within angle  $\beta$  of the  $z$  axis. For this figure,  $\alpha=20^\circ$  (hence  $\beta=21.6^\circ$ ), and the contour lines represent a 40-G surface for a baseball of radius  $R_B=1$  cm, current  $I_B=100$  A.

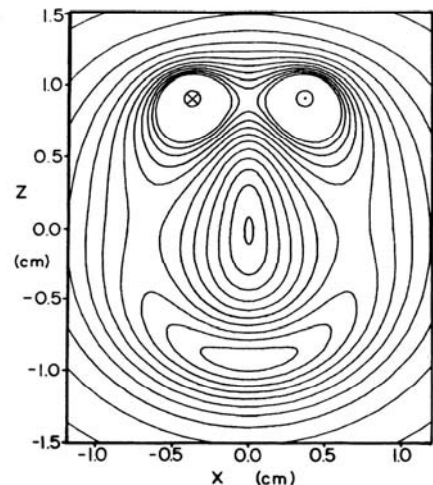
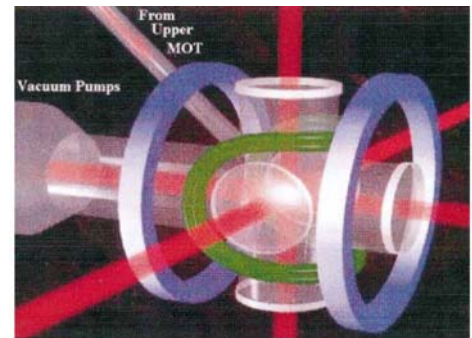


FIG. 10. Contours of  $|\mathbf{B}|$  in the  $x$ - $z$  plane of Fig. 9, with  $\alpha=20^\circ$ ,  $R_B=1$  cm, and  $I_B=100$  A. Contours are drawn at 10-G intervals. At the center,  $|\mathbf{B}|_{\min}=9.1$  G, while the transverse and axial saddle-point thresholds are 72 and 106 G, respectively.

# Quic Trap (LMU, ENS)

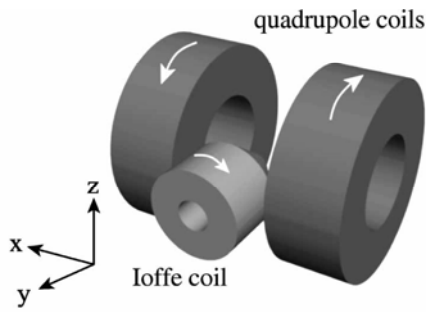
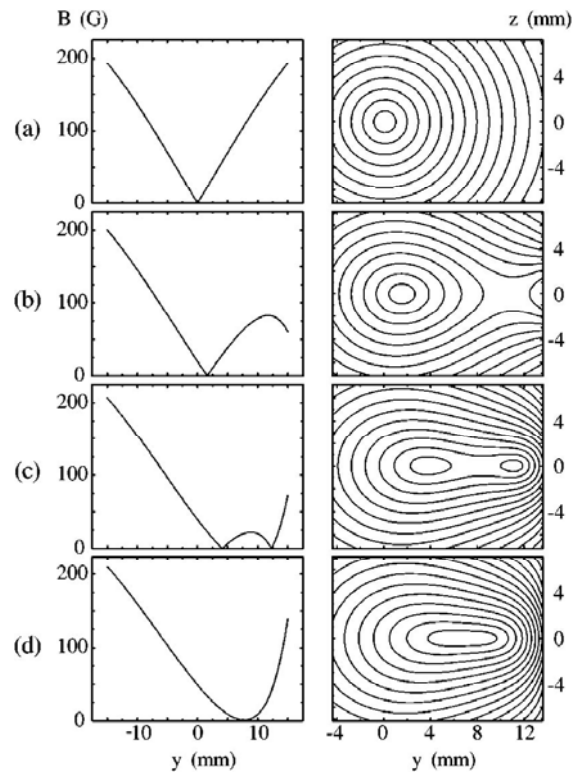
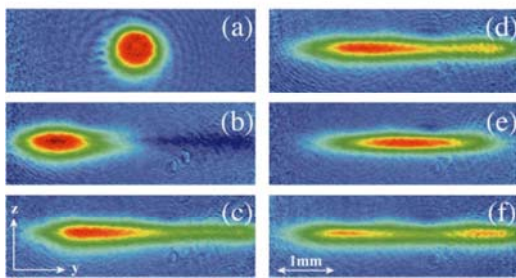


FIG. 1. QUIC trap: The Ioffe coil converts the spherical quadrupole trap into an Ioffe trap with its trap center close to the Ioffe coil. No additional coils are needed to produce a bias field. The arrows indicate the direction of the current flowing through the coils.



# Magnetic Transport

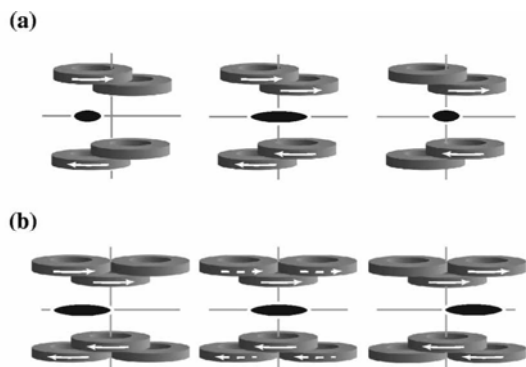


FIG. 2. Schematic setup for the transfer of a magnetic quadrupole trapping potential. The solid and dashed arrows indicate the current direction and strength. (a) By increasing the current in the second quadrupole coil pair and afterwards decreasing the current in the first quadrupole coil pair, the trapping potential may be moved. Here the aspect ratio is changed during the transport process. (b) By running suitable currents through three quadrupole coil pairs it is possible to maintain a constant aspect ratio during the transport process.

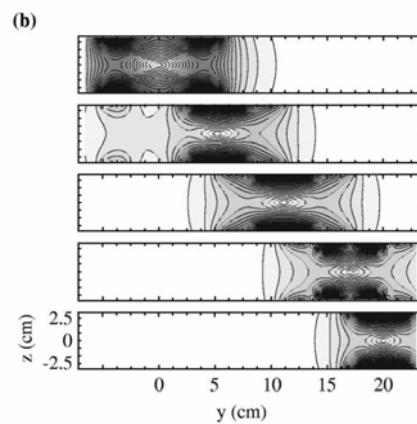
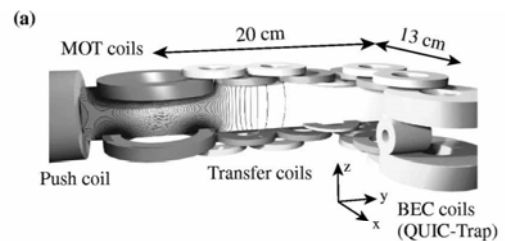
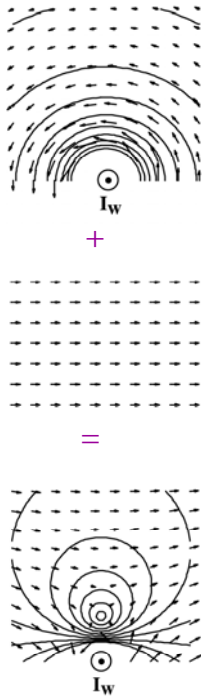


FIG. 1. (a) Experimental setup of the quadrupole coil pairs for the transport process. The magnetic trapping potential is moved over 33 cm around a 90° corner into an UHV vacuum region of a glass cell. (b) Contour plots, showing the absolute value of the magnetic field during different stages of the first half of the transport sequence.

# Micro Traps

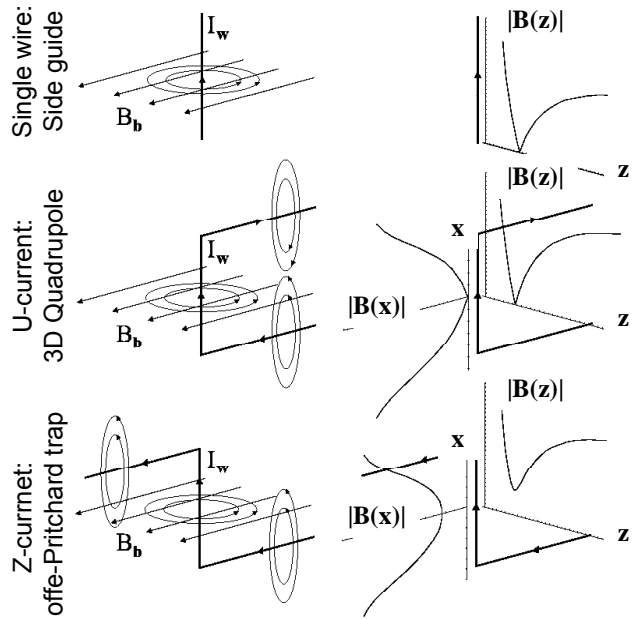


Trapping field is created by super-position of the field of a current carrying wire and a homogeneous bias field

Potential  
 depth: bias  
 field minimum: angle  
 current  $\leftrightarrow$  field  
 gradient:  $1/I$

Mount wire on a surface:  
 Use nanofabrication to build mesoscopic structures

## How to build a trap



Z-Wire Trap (Innsbruck)

# Paul Trap RF trap

Equation of motion:  $m \frac{d^2 u}{dt^2} + [a_u - 2q_u \cos(\Omega t)]u = 0$   
 (Mathieu equation)

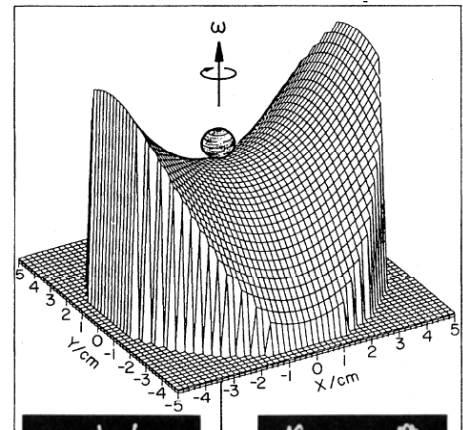
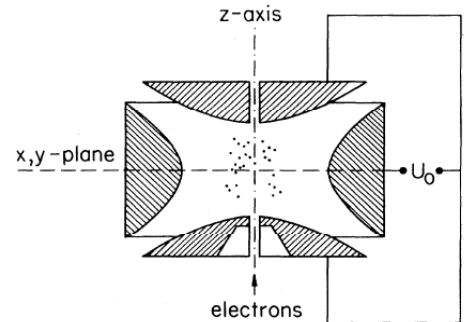
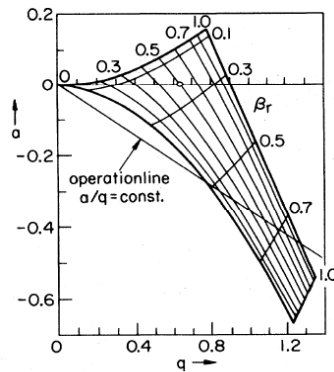
where  $u$  represents the  $x$ ,  $y$  and  $z$  coordinates, and  $a_u$  and  $q_u$  are dimensionless trapping parameters.

$$a_x = \frac{8eU}{mr_0^2 \Omega^2}$$

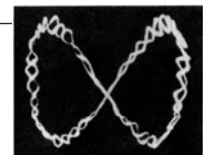
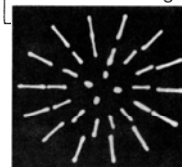
$$q_x = -\frac{4eV}{mr_0^2 \Omega^2}$$

$$a_z = -\frac{8eU}{mr_0^2 \Omega^2}$$

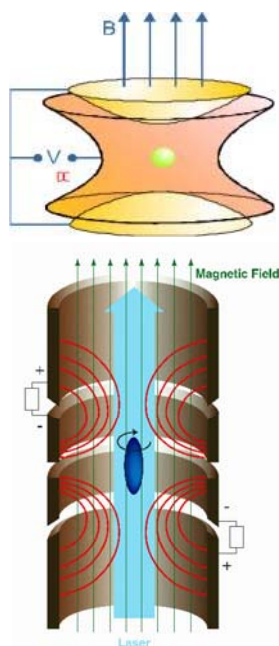
$$q_z = \frac{4eV}{mr_0^2 \Omega^2}$$



Mass spectrometers  
 Precision spectroscopy  
 Clocks  
 Quantum information



# Penning Trap



**Cyclotron oscillation:**

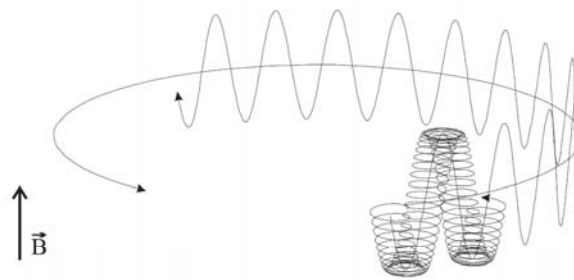
$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \approx \omega_c - \frac{\omega_z^2}{2\omega_c}$$

**Axial oscillation:**

$$\omega_z = \sqrt{\frac{1}{m} \frac{\partial^2 E_{\text{pot}}}{\partial z^2}(0)} = \sqrt{\frac{q C_2 V_0}{m d^2}}$$

**Magnetron oscillation:**

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \approx \frac{\omega_z^2}{2\omega_c}$$



$$\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2$$

$$\omega_c = \frac{q}{m} B_z \quad \text{mass, magnetic moments (g-2)} \quad \hbar\omega_L = -\mu B = -g m_S \mu_B B$$

## 5.2 Evaporative cooling to BEC

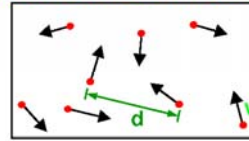
# BEC

## the basics

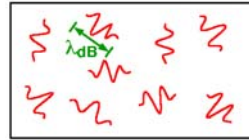
$$\frac{1}{e^{(E-\mu)/k_B T}} = A e^{-\frac{E}{k_B T}} \quad \text{classical particles Maxwell-Boltzmann statistics}$$

$$P(E) = \begin{cases} \frac{1}{e^{(E-\mu)/k_B T} - 1} & \mu < 0 \quad \text{Bosons Bose-Einstein statistics} \\ \frac{1}{e^{(E-\mu)/k_B T} + 1} & \mu = E_F \quad \text{Fermions Fermi-Dirac statistics} \end{cases}$$

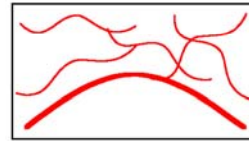
What is Bose-Einstein condensation (BEC)?



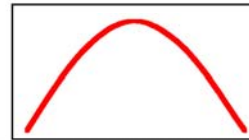
**High Temperature T:**  
thermal velocity  $v$   
density  $d^{-3}$   
"Billiard balls"



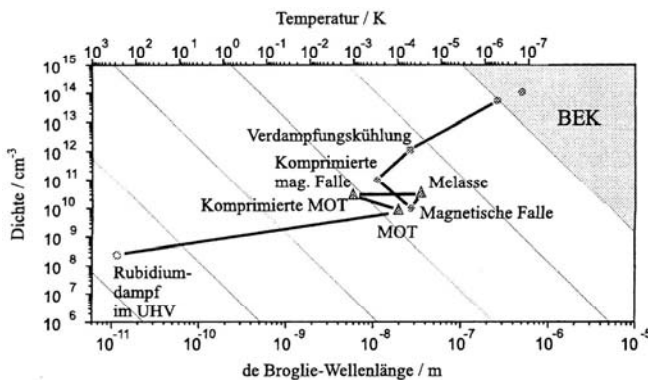
**Low Temperature T:**  
De Broglie wavelength  
 $\lambda_{dB} = h/mv \propto T^{-1/2}$   
"Wave packets"



**T=T<sub>crit</sub>:**  
Bose-Einstein Condensation  
 $\lambda_{dB} = d$   
"Matter wave overlap"



**T=0:**  
Pure Bose condensate  
"Giant matter wave"



## Two indistinguishable Particles

Wave function for 1 Particle  $\psi(x_1)$   
 Wave function for 2 Particles  $\psi(x_1, x_2)$   
 We can only observe  $|\psi(x_1, x_2)|^2$

If the particles are indistinguishable we find:  $|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$

There are 2 possibilities:

Boson :  $\psi_+(x_1, x_2) : \psi_+(x_1, x_2) = +\psi_+(x_2, x_1)$   

$$\psi_+(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) + \psi_2(x_1)\psi_1(x_2)]$$

Fermion  $\psi_-(x_1, x_2) : \psi_-(x_1, x_2) = -\psi_-(x_2, x_1)$   

$$\psi_-(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)]$$



# Two indistinguishable Particles II

sets see what happens if the two particles are at the same location (state)

that is if  $x_1 = x_2$

$\psi_+(x_1, x_1) \neq 0 \rightarrow$  Bosons can occupy the same state

$\psi_-(x_1, x_1) = 0 \rightarrow$  Fermions can not occupy the same state

consequently for 2 Bosons at the same location (in the same state) we find:

$$|\psi(x, x)|^2 = 2 |\psi_1(x)\psi_2(x)|^2$$

probabilities to find n particles in the same state:

n classical particles  $P_n = (P_1)^n$

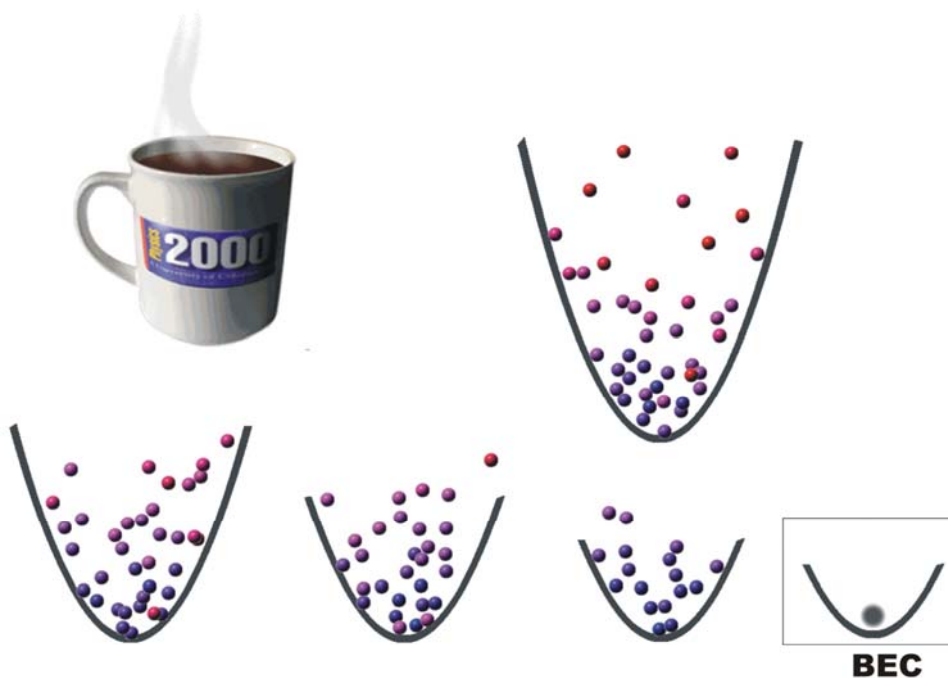
n Bosons  $P_n^{Boson} = n!(P_1)^n$

probability to add another Boson to a state with n Bosons

$$P_{n+1}^{Boson} = (n+1)P_1 P_n^{Boson}$$

stimulated scattering, stimulated emission

# Cooling in a Conservative Trap evaporative cooling

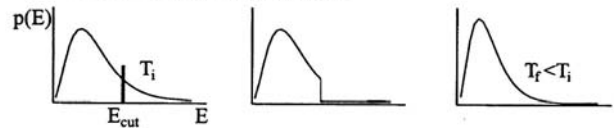


<http://www.Colorado.EDU/physics/2000/bec/>

# Evaporative Cooling

H.F. Hess Phys.Rev.B 34 p R3476 (1986)  
 K.B. Davis, M.-O. Mewes, W. Ketterle Appl.Phys.B 60, p155 (1995)  
 O.J.Luiten, M. Reynolds, J. Walraven Phys.Rev.A 53 p381 (1996)

## Schrittweise Verdampfungskühlung:



## General Scaling

Evaporative cooling happens at exponential scale  
 Time scale is given by the thermalisation (collision) time  
 (it takes about 5 collision for a truncated distribution to thermalise)

Characteristic quantities are logarithmic derivatives  
 A key parameter is  $\alpha = \frac{d(\ln T)}{d(\ln N)}$  (temperature decrease per particle loss)  
 Evaporation is controlled by a potential depth  $\eta kT$

The density of states ( $\rho(\epsilon)$ ) of trap pot. determines scaling.

For power law potentials:  $\rho(\epsilon) = A_{pl} \epsilon^{1/2+\delta}$   
 square well:  $\delta=0$ , harmonic:  $\delta=3/2$ , linear:  $\delta=3$

Ioffe-Pritchard trap:  $\rho(\epsilon) = A_{IP} (\epsilon^3 + 2U_{IP} \epsilon^2)$

Exponential scaling for a d-dimensional potential  $U(r) \sim r^{d/\delta}$

Number of atoms N	1	with: $\alpha = \frac{d(\ln T)}{d(\ln N)} = \frac{\eta + \kappa}{\delta + \frac{1}{2}} - 1$
Temperature T	$\alpha$	
Volume V	$\alpha \delta$	average energy in potential $\langle \epsilon \rangle = (\delta + 3/2) kT$
Density n	$1 - \alpha \delta$	
Phase space density D	$1 - \alpha(\delta + 3/2)$	
Elastic coll. Rate $\nu_{ev}$	$1 - \alpha(\delta - 1/2)$	

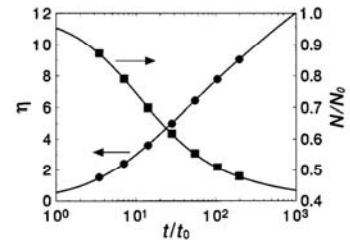


FIG. 5. Truncation parameter  $\eta$  (circles) and fraction of atoms remaining in trap  $N/N_0$  (squares) as a function of reduced time  $t/t_0$  after initiating evaporation from infinite temperature. Curves are obtained by integration of the differential equations resulting from the truncated Boltzmann approximation, symbols by fitting to the distribution obtained by numerical solution of the kinetic equation.

## Run-away evaporation:

Achieve faster and faster thermalisation with cooling

Collision rate has to grow:  $\alpha(\delta - 1/2) > 1$

Collisions:

good collisions: elastic collisions  
 bad collisions: inelastic coll., trap loss coll., etc ... limit the trap lifetime

How many collisions per tapping time for run away evap.?

Linear trap: > 25 collisions per lifetime  
 Harmonic trap: > 150 collisions per lifetime

Atoms – Light and Matter Waves J. Schmiedmayer, A. Rauschenbeutel

## RF induced evaporation

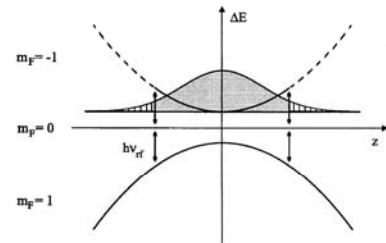


Abbildung 3.11: Radiofrequenz-induzierte Verdampfungskühlung in einem magnetischen Potential.

[r.]

# Evaporative Cooling

experiment

## Methods of evaporation

First experiments (Hydrogen):

contact with walls Hess et al. PRL 59, 672 (1987)  
 Saddle point evaporation Masuhara et al. PRL 61, 935 (1988)

RF induced evaporation

Na Davis et al. PRL 74, 5202 (1995)  
 Rb Petrich et al. PRL 74, 3352 (1995)

Optical traps by lowering the potential

Na Adams et al. PRL 74, 3577 (1995)

## Dimensionality of evaporation:

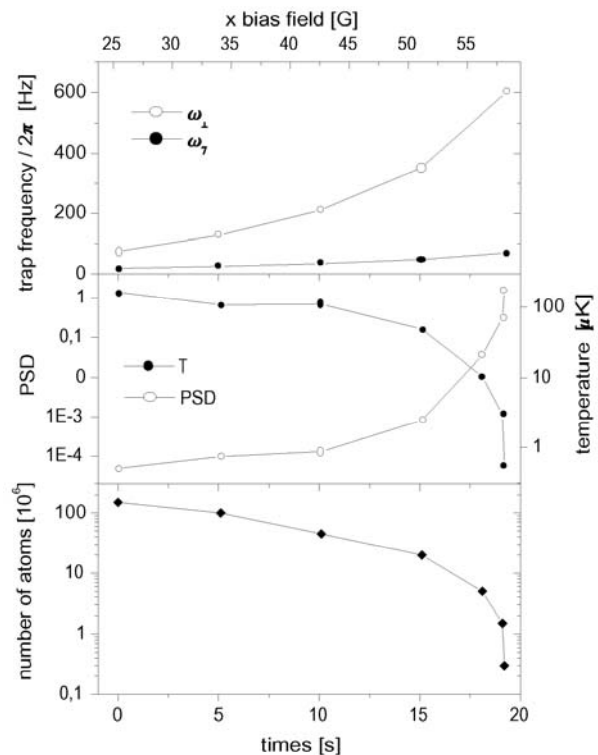
To achieve fast and efficient evaporation the loss mechanism should be open or all directions of motion in the trap (3-d evaporation)

Lower dimensional evaporation (2-d or 1-d) require effective mixing of the degrees of freedom in the motion inside the trap.

## Heating from a shaking trap:

Simple model:  $\frac{1}{t_{heat}} = \pi^2 v_{trap}^2 S(2v_{trap})$

$S(v) = \epsilon^2 / \Delta v$  is the power spectrum of the fractional field noise  $\epsilon$  in a bandwidth  $\Delta v$   
 stability of  $10^{-3}$  to  $10^{-4}$  is adequate to achieve BEC



# Observing the BEC

## Methods of imaging:

3 processes: absorption, emission, shifting the phase  
 3 methods: absorptive, fluorescence, dispersive imaging  
 Description: complex index of refraction

$$n_{ref} = 1 + \frac{\sigma_0 n \lambda}{4\pi} \left[ \frac{i}{1+\delta^2} - \frac{\delta}{1+\delta^2} \right] \quad \text{with } \delta = \frac{\omega - \omega_0}{\Gamma/2}$$

Transmission T and phase shift  $\Phi$

$$T = e^{-\tilde{D}/2} = \exp\left(-\frac{1}{2} \frac{\tilde{n} \sigma_0}{1+\delta^2}\right) \quad \text{with } \tilde{D} = \frac{\tilde{n} \sigma_0}{1+\delta^2}$$

$$\Phi = -\delta \frac{\tilde{D}}{2} = -\frac{\delta}{2} \frac{\tilde{n} \sigma_0}{1+\delta^2}$$

## Imaging dense clouds ( $D_0 > 100$ ):

Optimal absorption imaging is at optical density of 1  
 Need large detuning but there is refraction at  $|\delta| > 0$   
 For diffraction limited imaging we need a phase shift  $\Phi < \pi/2$   
 For optical density 1 we need  $|\delta| = (D_0)^{1/2}$   
 At this detuning the phase shift  $\Phi \sim 0.5 (D_0)^{1/2}$  much too large  
 For  $\Phi < \pi/2$  we need  $|\delta| = D_0/\pi$   
 Need phase contrast imaging

## Non destructive imaging:

Example of an image:  $30 \times 30$  pixel with 100 photons each ( $10^5$  photons)  
 This can be 'non perturbative' for large condensates.

Important figure of merit: ratio signal / heating

**Absorption imaging:** each photon gives one recoil energy

**Dispersive imaging:** there are more forward scattered photons than absorbed photons  
 for large detuning the gain is  $D_0/4$  !  
 elastic scattered photons contribute not to heating if imaging is done  
 in the trap and light pulse is longer than  $1/V_{trap}$   
 one can make many pictures of the condensate

For low density clouds there is no advantage of dispersive imaging

Atoms – Light and Matter Waves

J. Schmiedmayer, A. Rauschenbeutel

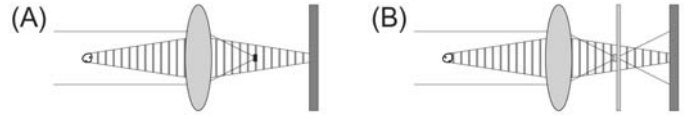
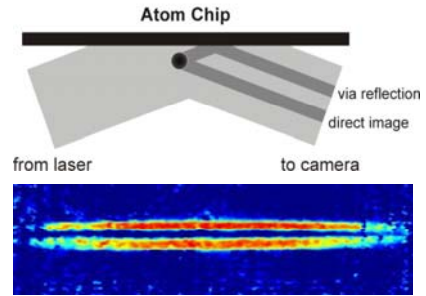
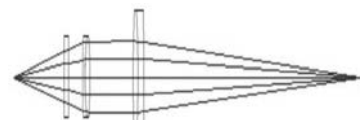


Fig. 7. – Dark-ground (A) and phase-contrast (B) imaging set-up. Probe light from the left is dispersively scattered by the atoms. In the Fourier plane of the lens, the unscattered light is filtered. In dark-ground imaging (A), the unscattered light is blocked, forming a dark-ground image on the camera. In phase-contrast imaging (B), the unscattered light is shifted by a phase plate (consisting of an optical flat with a  $\lambda/4$  bump or dimple at the center), causing it to interfere with the scattered light in the image plane.



Lens setup ( $3\mu\text{m}$  resolution)

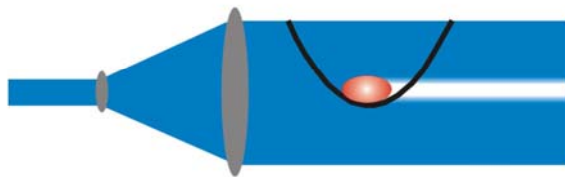


Spot diagram

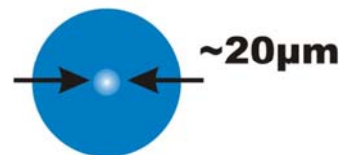
Lecture 4 <Nr.>

# Wie sieht man die Kondensation

von der Seite

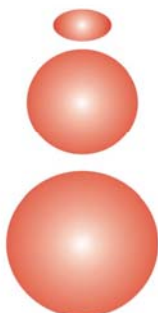


von vorne

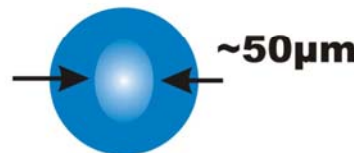
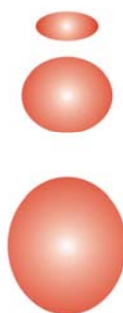


Freie Expansion

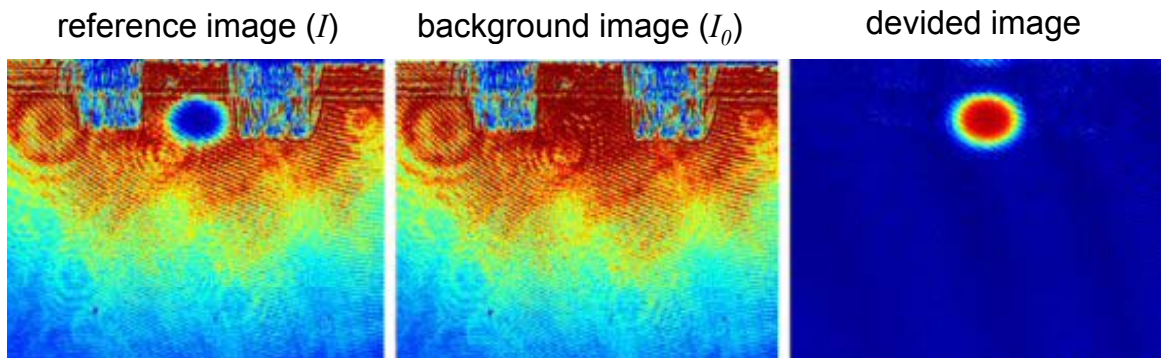
thermisch



BEC



# absorption imaging



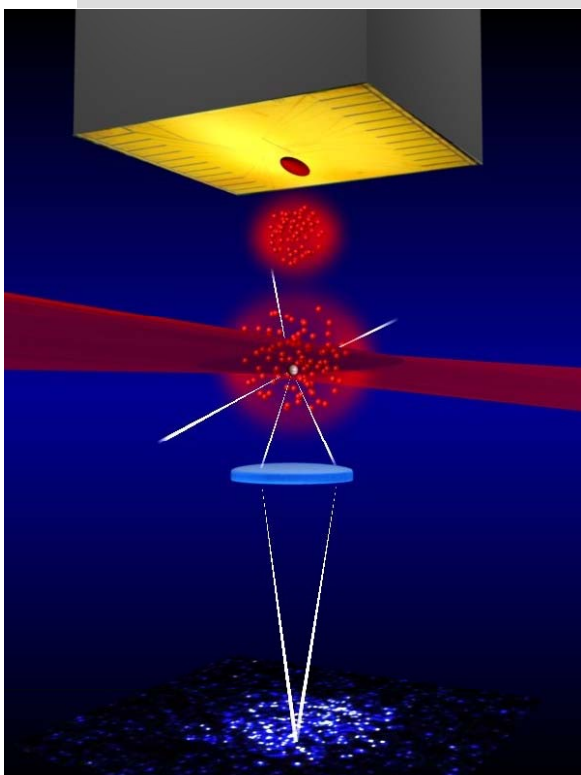
Typical example: an image with  $30 \times 30$  pixels and 100 detected photons/pixel involve  $10^5$  absorbed photons

Heating:  $1E_{rec}$  per photon  $\rightarrow$  ejects atom from BEC

absorption imaging is **destructive** for BECs  $\leq 10^5$  atoms,

larger BECs can be imaged several times (H-BEC:  $10^9$  atoms)

# A single-atom fluorescence camera

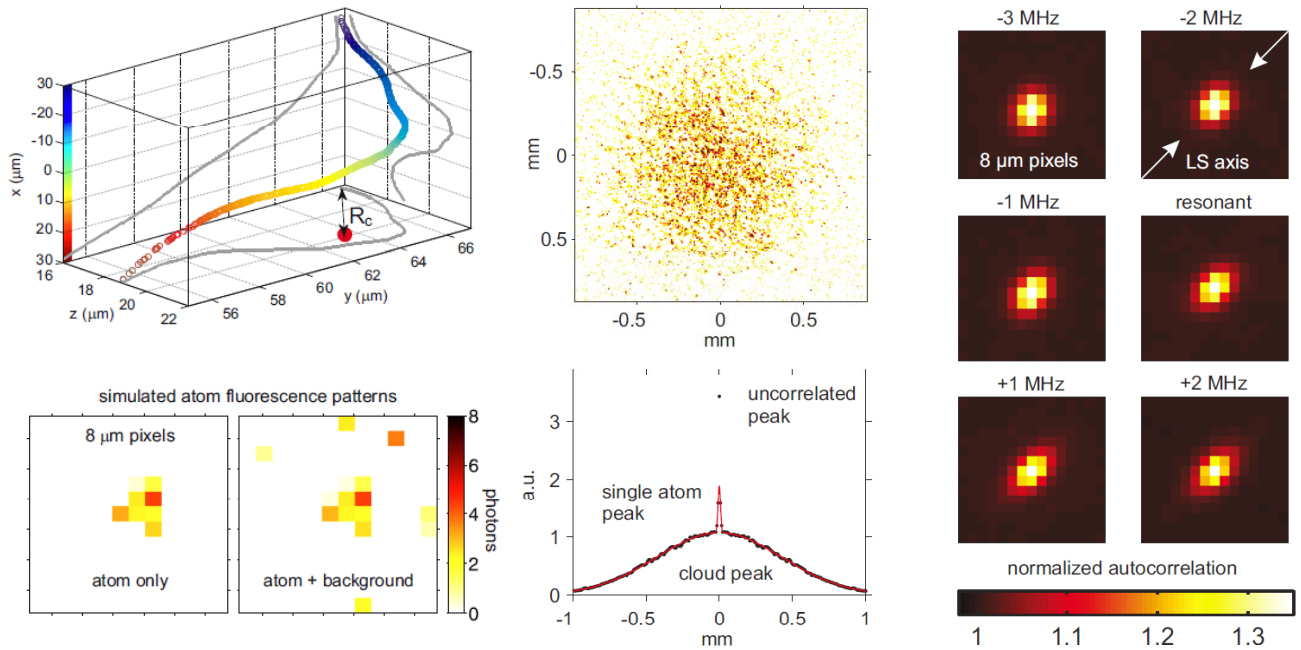


## Experimental realization

- Atom cloud is released from **chip trap**
- After 4 – 8 mm fall it passes a thin **sheet of light**:
  - two counter-propagating lasers,  $20 \mu\text{m}$  waist
  - resonant / detuned from  $|5S_{1/2}, F=2\rangle \rightarrow |5P_{3/2}, F=3\rangle$
  - in light sheet, each atoms scatters  $\sim 900$  photons
- a **high NA imaging system** captures 20 photons
  - numerical aperture 0.34
  - depth of field  $40 \mu\text{m}$  (essentially zero background)
  - spatial resolution  $8 \mu\text{m}$  over 4 mm diameter
- **intensified (EMCCD) camera** records images
  - major noise source: clock induced charges  $\cong 1$  photon

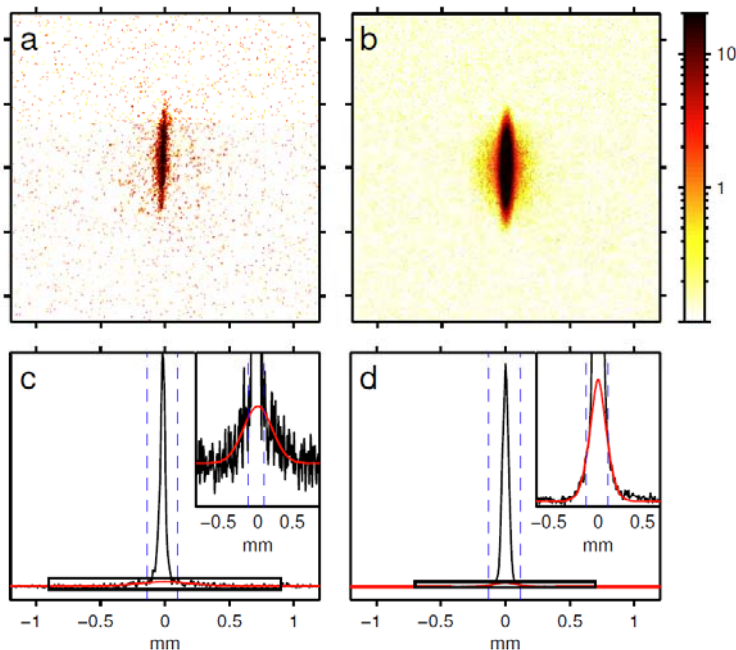
# Fluorescence Camera

## Single Atom Detection



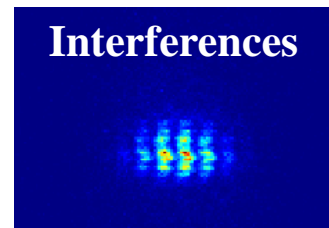
# Fluorescence Camera

## Large Dynamic Range



The large dynamic range of the single atom sensitive imaging allows to see very small thermal clouds besides the BEC, and measure very small temperatures  $T \sim T_c/10$

**Interferences**



# imaging techniques - a comparison

All imaging techniques equally illuminate the atom cloud with a probe beam. They differ in how much information can be extracted per absorbed photon  
 → figure of merit ( $\zeta$ )

$$\frac{\zeta_{\text{dispersive}}}{\zeta_{\text{absorptive}}} \approx \frac{D}{4}$$

Best suited imaging:

- dense clouds/BECs ( $D \cong 300$ ):
- dilute clouds ( $D \cong 1$ ):
- single atoms ( $D \ll 1$ )

**dispersive** (phase contrast) **imaging** allows to take up to 100 frames of a single sample (in-situ or short ToF)

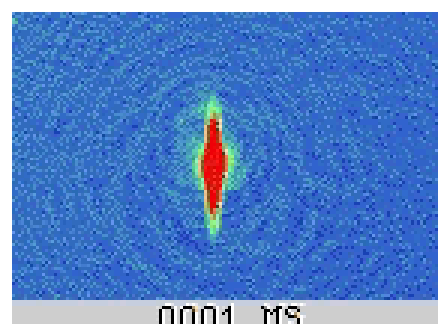
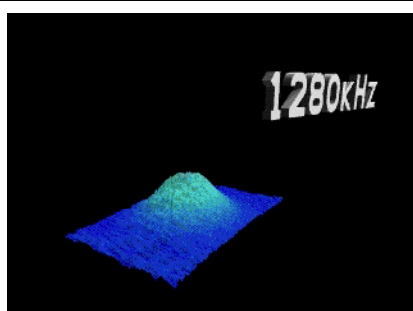
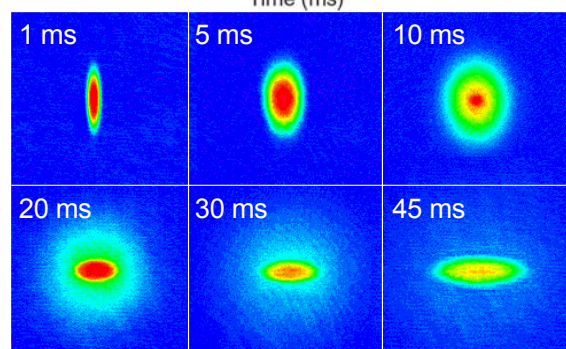
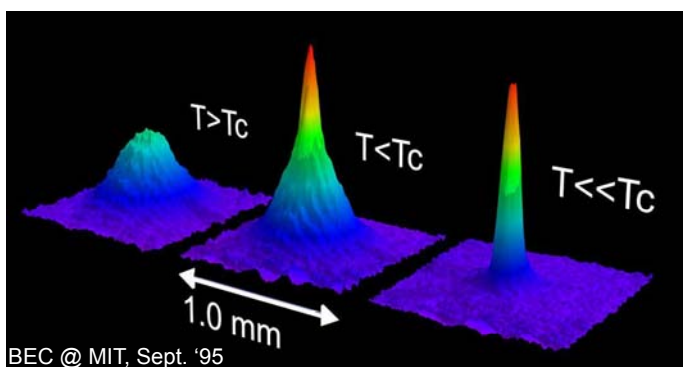
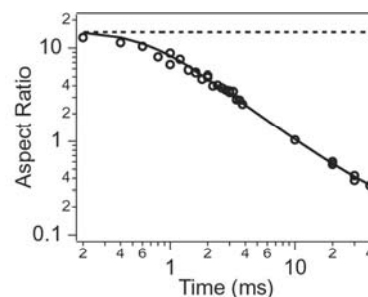
**absorption imaging** allows to detect down to 10 atoms/ $\mu\text{m}^2$  for long ToFs (problematic when imaging dense clouds)

in **fluorescence imaging**, the signal/noise ratio is determined by

- electronic noise (<2 photons /pixel)
  - background light reaching the detector
- single atom detection is possible when reducing background light to  $10^{-5}$

Remark: single atom detection is **easy** using ionizing / MCP detection (→ **seminar**)

## Observing BEC



# Quantum Degenerate Bose and Fermi Gas

Andrew G. Truscott et al. Science 291, p2570 (2001)

**Bosonen**

gehen in den Grundzustand

**Fermionen**

füllen alle Zustände auf

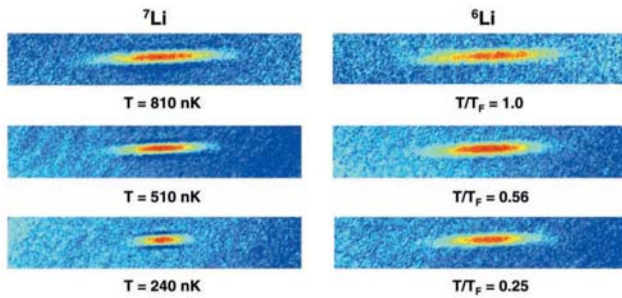


Fig. 1. Two-dimensional false-color images of both  $^7\text{Li}$  and  $^6\text{Li}$  clouds. At  $T/T_c = 1.0$ , the two clouds are approximately the same size, but as the atoms are cooled further, to  $T/T_c = 0.56$ , the Bose gas contracts, whereas the Fermi gas exhibits only subtle changes in size. At  $T/T_c = 0.25$ , the size difference between the two gases is clearly discernible. At this temperature the  $^7\text{Li}$  image displays distortions due to high optical density. However, these distortions are present only in the radial direction and do not affect the measurements. The fitted numbers of  $^7\text{Li}$  and  $^6\text{Li}$  atoms,  $N_7$  and  $N_6$ , and the fitted temperatures are as follows: For the upper set,  $N_7 = 2.4 \times 10^5$ ,  $N_6 = 8.7 \times 10^4$ , and  $T = 810 \text{ nK}$ ; for the middle set,  $N_7 = 1.7 \times 10^5$ ,  $N_6 = 1.3 \times 10^5$ , and  $T = 510 \text{ nK}$ ; and for the lower set,  $N_7 = 2.2 \times 10^4$ ,  $N_6 = 1.4 \times 10^5$ , and  $T = 240 \text{ nK}$ . The probe detuning is a parameter of the fits but is constrained to vary by no more than its uncertainty of  $\pm 3 \text{ MHz}$ . The fits result in typical reduced- $\chi^2$  values of  $\sim 1.0$ . The uncertainties in number and temperature are due mainly to the uncertainties in the fit and are roughly estimated by finding the point at which the reduced- $\chi^2$  increases by 20%. The resulting uncertainties are 8% in temperature and 15% in number. Other sources of uncertainty are relatively insignificant. The size of each displayed image is 1.00 mm in the horizontal axis and 0.17 mm in the vertical axis.

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Lecture 4 <Nr.>

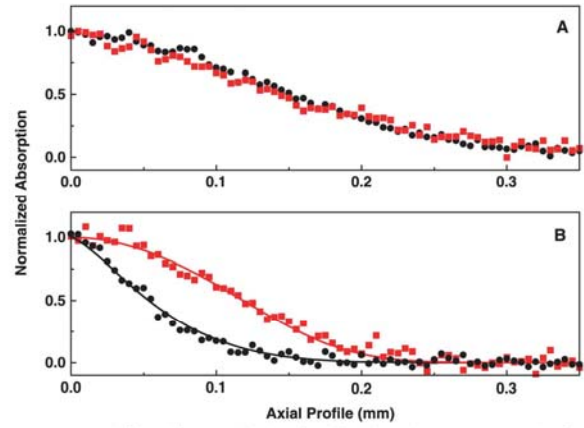
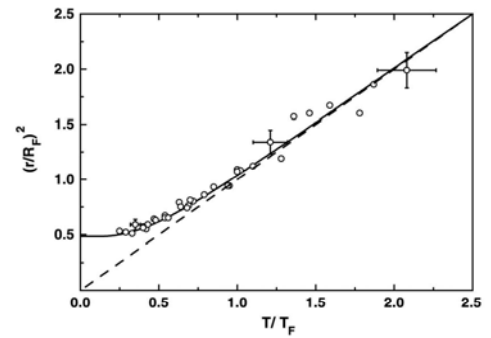


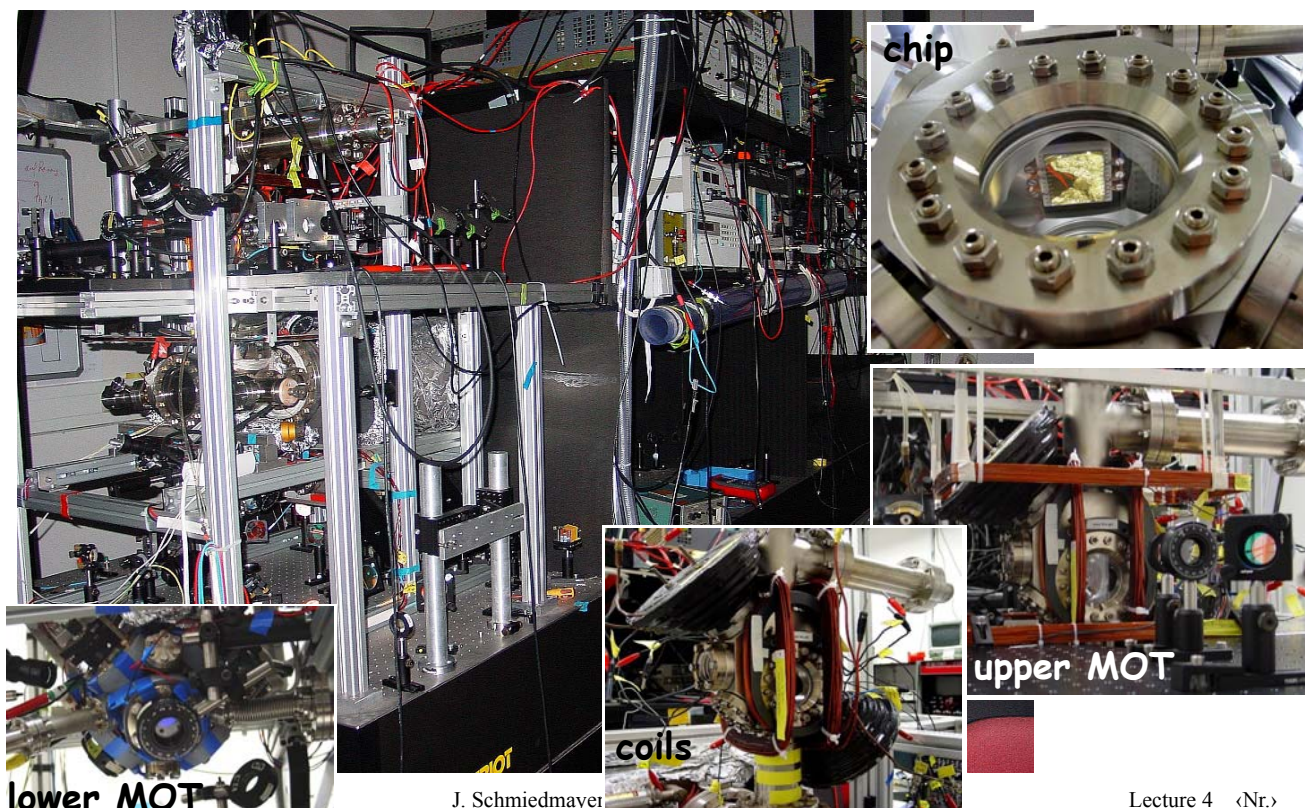
Fig. 2. Comparison of  $^6\text{Li}$  and  $^7\text{Li}$  atom cloud axial profiles. The red squares correspond to  $^6\text{Li}$ , and the black circles to  $^7\text{Li}$ . (A) Data from the top image of Fig. 1, corresponding to  $T/T_c = 1.0$  and  $T/T_c = 1.5$ . (B) Data from the lower image of Fig. 1, corresponding to  $T/T_c = 0.25$  and  $T/T_c = 1.0$ . The fits to the data are shown as solid lines.

Fig. 3. Square of the  $1/e$  axial radius,  $r$ , of the  $^6\text{Li}$  clouds versus  $T/T_c$ . The radius is normalized by the Fermi radius,  $R_F = (2k_B T_c / m\omega_z^2)^{1/2}$ , where  $m$  is the atomic mass of  $^6\text{Li}$ . The solid line is the prediction for an ideal Fermi gas, whereas the dashed line is calculated assuming classical statistics. The data are shown as open circles. The divergence of the data from the classical prediction is the result of Fermi pressure. Several representative error bars are shown.



These result from the uncertainties in number and temperature as described in the legend to Fig. 1. In addition, we estimate an uncertainty of 3% in the determination of  $r$ .

## VACUUM CHAMBER - REALITY



J. Schmiedmayer

Lecture 4 <Nr.>

## 5.3 RF induced adiabatic potentials

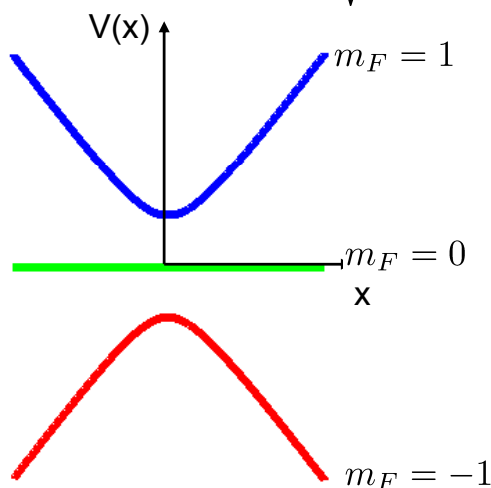
### Combining static and RF fields

#### Ioffe-Pritchard trap

$$\mathbf{B}_S(\mathbf{r}) = Gx\mathbf{e}_x - Gy\mathbf{e}_y + B_I\mathbf{e}_z$$

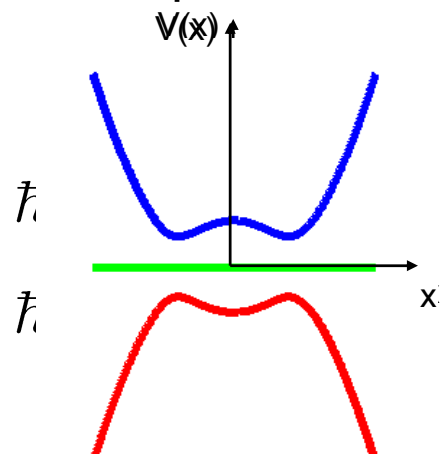
$$V_{\text{ad}}(r) = g_F\mu_B F_z |\mathbf{B}_S(\mathbf{r})|$$

$$= g_F\mu_B m_F \sqrt{G^2\rho^2 + B_I^2}$$



- in a source-free region only mag. field minima are achievable
- number of possible trap shapes can be greatly increased by adding an oscillating RF magnetic field

#### dressed RF potentials



Zobay & Garraway PRL **86**, 1195 (2001)  
I. Lesanovsky et al. PRA **73** 033619 (2006)



# Dressed adiabatic potentials

## Oscillating RF magnetic field

$$\mathbf{B}_{RF}(\mathbf{r}, t) = \frac{B_{RF}}{\sqrt{2}} [\mathbf{e}_x \cos(\omega t) + \mathbf{e}_y \cos(\omega t + \delta)]$$

## Total Hamiltonian

$$H = \frac{\mathbf{p}^2}{2M} + g_F \mu_B \mathbf{F} \cdot [\mathbf{B}_S(\mathbf{r}) + \mathbf{B}_{RF}(\mathbf{r}, \omega t)]$$

relative phase shift

1. apply the unitary transformation  $U_S(\mathbf{r})$  to diagonalize the static part
2. transform into a rotating frame around the local quantization axis
3. perform the rotating-wave-approximation
4. diagonalize spin-field interaction terms

$$H_{\text{final}} = \frac{1}{2M} [\mathbf{p} + \mathbf{A}(\mathbf{r}, t)]^2 - \frac{1}{2M} \Phi(\mathbf{r}, t) + g_F \mu_B |\mathbf{B}_{\text{eff}}(\mathbf{r})| F_z$$

adiabatic approximation      dressed adiabatic potentials

$\mathbf{B}_{\text{eff}}$  does not necessarily obey Maxwell's equations

- potential depends on the relative orientation of the RF and the static field
- **spatial dependence** gives rise to novel types of RF traps
- free parameter  $\delta$ , i.e. RF polarization can be used to modify the trap shape

theory: I. Lesanovsky et al. PRA **73** 033619 (2006)  
I. Lesanovsky et al. PRA **74** 033619 (2006)  
Atoms – Light and Matter Waves

experiment: T. Schumm et al. Nature Physics **1**, 57 (2005)  
S. Hofferberth et al. Nature Physics **2**, 710 (2006)  
Lecture 4 (Nr.)

# RF induced Potentials

## state dependent potentials by RF polarization

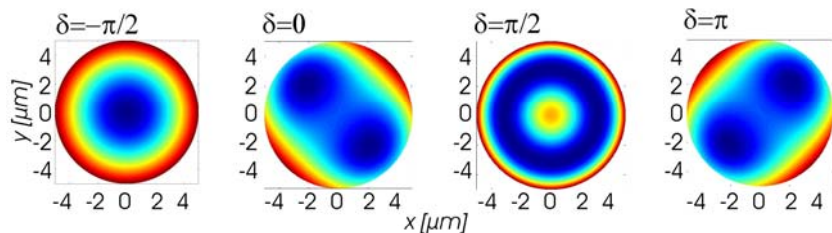
**Polarization** of the RF field gives extra freedom

$$\mathbf{B}_S(\mathbf{r}) = Gx\mathbf{e}_x - Gye_y + B_I\mathbf{e}_z$$

$$\mathbf{B}_{RF}(\mathbf{r}, t) = \frac{B_{RF}}{\sqrt{2}} [\mathbf{e}_x \cos(\omega t) + \mathbf{e}_y \cos(\omega t + \delta)]$$

$$V_{\text{ad}}(\mathbf{r}) = m_F g_F \mu_B \sqrt{\Omega^2(\mathbf{r}) + \Delta^2(\mathbf{r})}$$

tuning the relative RF phase  $\delta$



$$\Omega(\mathbf{r}) = |\mathbf{B}_S(\mathbf{r})| - \frac{\hbar\omega}{g_F \mu_B}$$

$$\Delta(\mathbf{r}) = \frac{B_{RF}}{2} \left[ 1 + \frac{B_I \sin \delta_{\text{eff}}}{|\mathbf{B}_S(\mathbf{r})|} + \frac{G^2 \rho^2}{2 |\mathbf{B}_S(\mathbf{r})|^2} (\cos \delta_{\text{eff}} \sin(2\phi) - 1) \right]$$

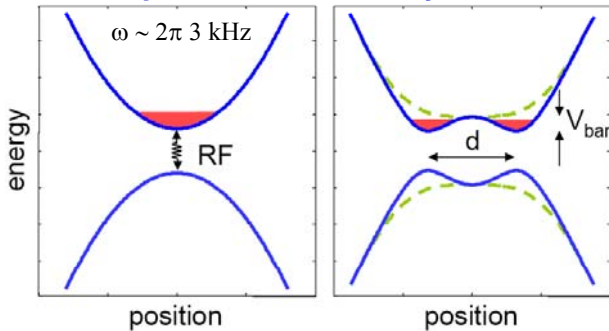
state dependent

$$\delta_{\text{eff}} = \frac{g_F}{|g_F|} \delta$$

# RF-Dressed State Potentials

## Creating a Double Well

### Couple atomic states by RF / MW



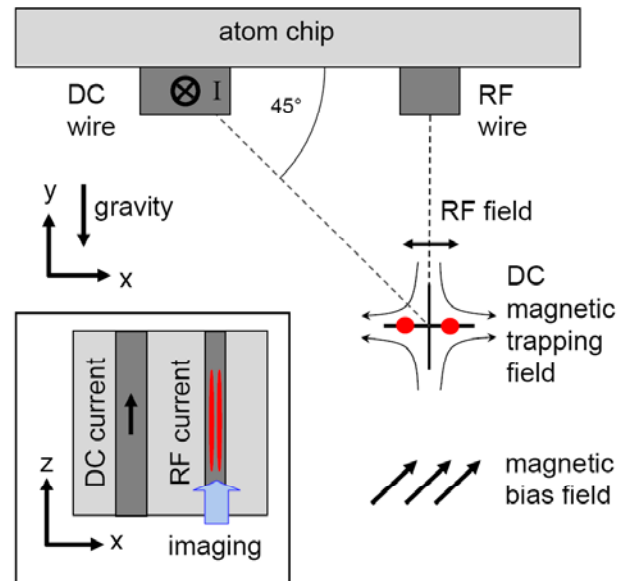
- The minimum of the adiabatic potential is at iso-B surfaces
- The minimum value at the iso-B surface depends on the **RF** coupling strength
- The coupling strength depends on the **orientation** of the RF field relative to the trap field

$$V_{\text{ad}}(\mathbf{r}) = m_F g_F \mu_B \sqrt{\Omega^2(\mathbf{r}) + \Delta^2(\mathbf{r})}$$

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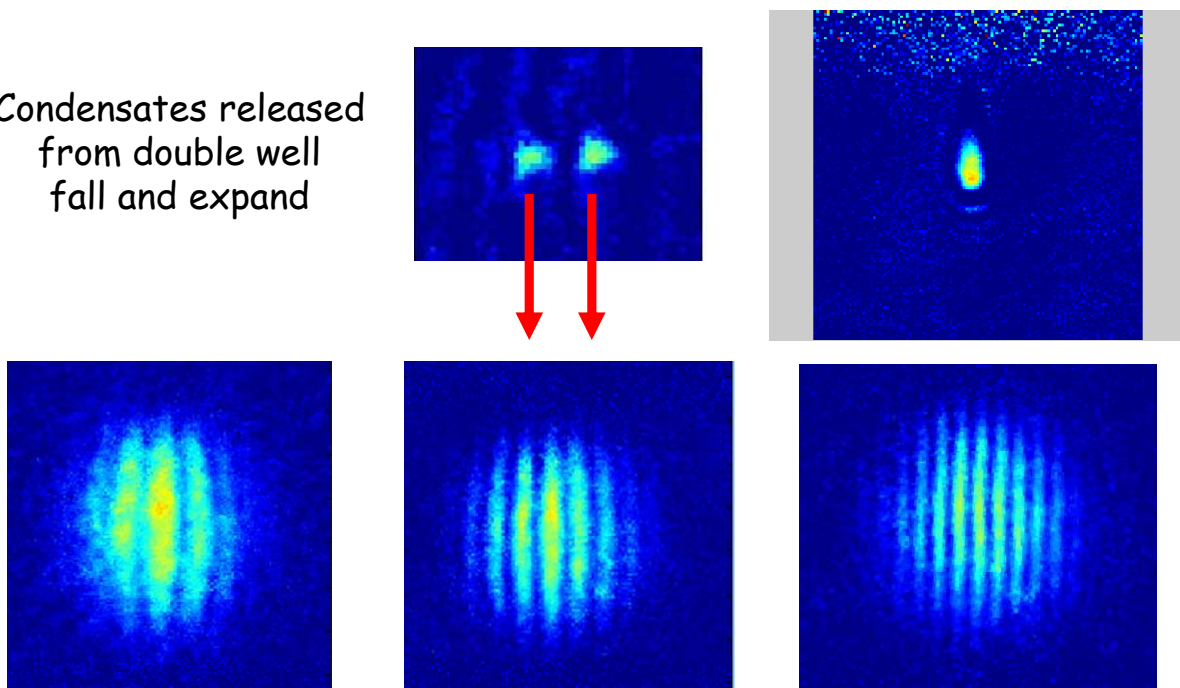
### realization on AtomChip



'Matter-wave interferometry in a double well on an atom chip',  
T. Schumm, et al., Nature Physics. 1, 57 (2005)

# Observe interference in time of flight

Condensates released from double well fall and expand



Interference between overlapping BECs

Atoms – Light and Matter Waves

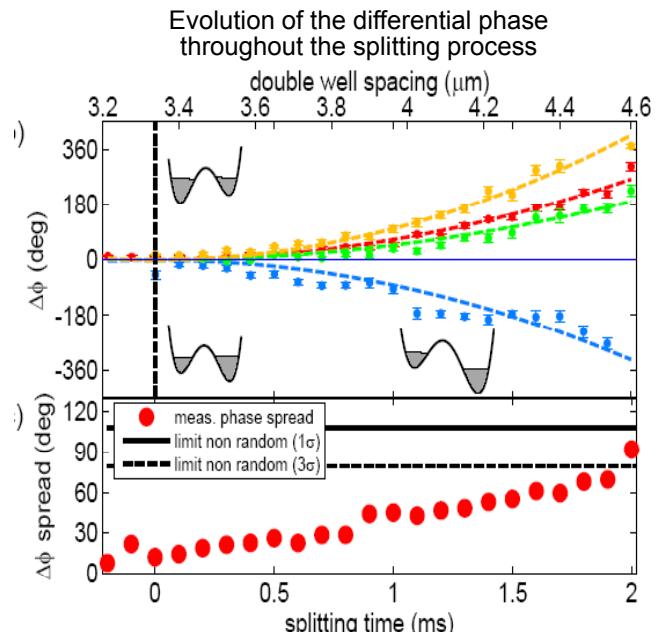
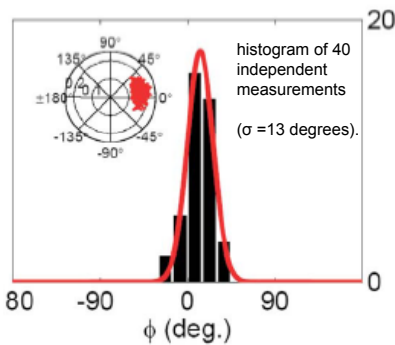
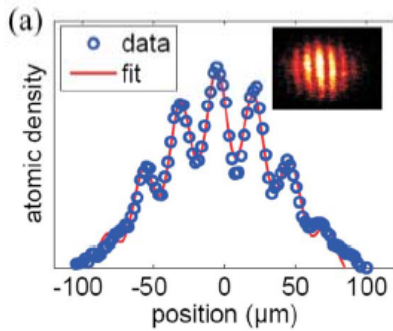
J. Schmiedmayer, A. Rauschenbeutel

TOF = 16ms

Lecture 4 <Nr.>

# Coherent Splitting

After the BECs has been split far enough to inhibit tunneling ( $d=3.4 \mu\text{m}$ ), atoms are released and an interference pattern is observed after a time of flight.



'Matter-wave interferometry in a double well on an atom chip', T. Schumm, et al., Nature Physics. 1, 57 (2005)

Atoms – Light and Matter Waves

J. Schmiedmayer, A. Raus

# State-dependent double well interference

Demonstration with  $F=2, m_F=2$  state of  $^{87}\text{Rb}$

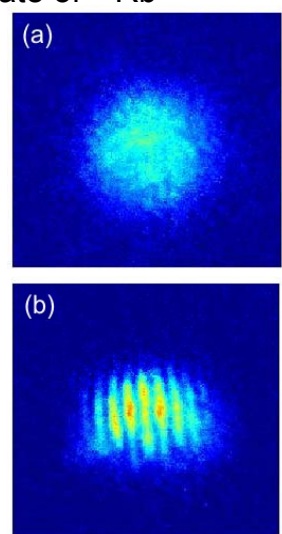
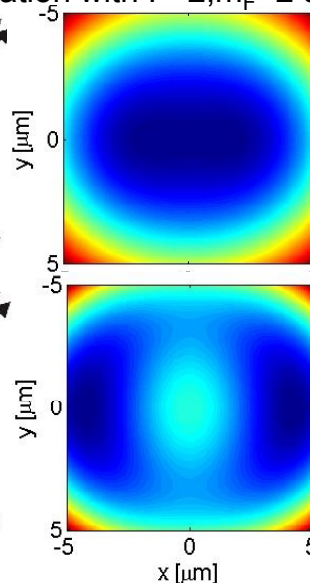
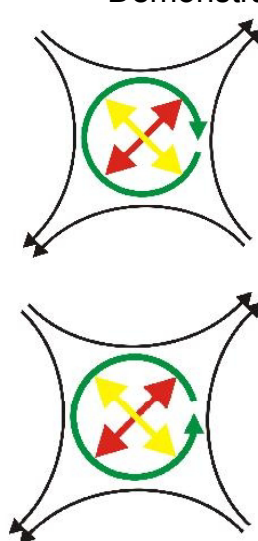
„effective“ polarization depends on  $g_F$

$$\delta_{\text{eff}} = \frac{g_F}{|g_F|} \delta$$

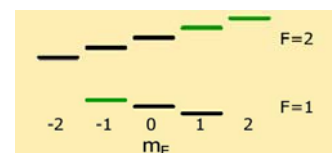
RF-potentials allow state-dependant manipulation

elliptical polarization:

state-dependant double well



possible application: collisional phase gate



Hofferberth et al. Nature physics Oct. (2006)  
Atoms – Light and Matter Waves

J. Schmiedmayer, A. Rauschenbeutel

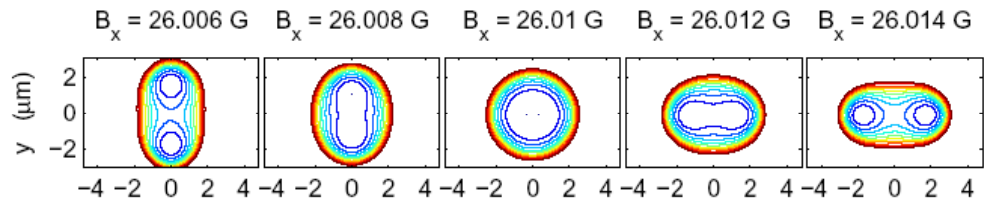
Lecture 4 <Nr.>

# Advantages of RF potentials splitting a trap

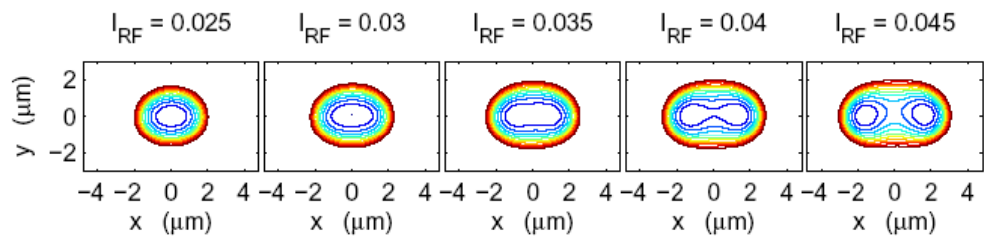
Hoffererth et al. Nature Physics 2, 710 (2006)

- True splitting 1 trap -> 2 traps
  - Confinement in transvrsl direction stays the same
  - Confinement in splitting direction is significantly tighter
- splitting potential:  $V(x)=A(t) x^2+B x^4$   
the size of the  $x^4$  term determines the confinement  
In RF potentials **B** is factor ~1000 larger

2-wire  
beam splitter  
contours 5 Hz



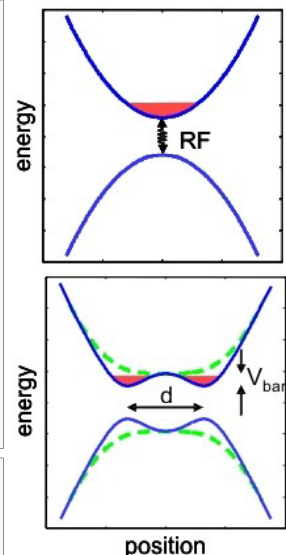
RF  
beam splitter  
contours 5 kHz



# RF and MW induced adiabatic potentials

create adiabatic dressed state potentials by coupling electronic ground states of an atom

- coupling between stable states allows to create conservative potentials even with **on resonant radiation**
- shaping the potential:
  - **detuning** the states with an external magnetic field
  - **spatial dependent coupling strength** (RF field)  
-> allows strong field seeker traps
- coupling is magnetic:  
the **amplitude** and the relative **orientation** of the RF field and the detuning field are important



- first experiment: dressed neutrons: E. Muskat et al., PRL **58**, 2047 (1987).
- first proposal of a MW trap (detuned): C. Agosta, et al. PRL **62**, 2361 (1989).
- MW experiment (Cs, detuned): R. Spreuw, et al. PRL **72**, 3162 (1994).
- RF dressed state traps (with magnetic field detuning but neglecting polarization): O. Zobay, B. M. Garraway, PRL **86**, 1195 (2001).
- RF potentials for thermal Rb atoms: Y. Colombe, et al. Europhys. Lett. **67**, 593 (2004).
- Full implementation: T. Schumm et al Nature Physics **1**, 57 (2005)

# RF and MW induced state dependent potentials

The two clock states have

$$|F = 2, m_F = 1\rangle$$

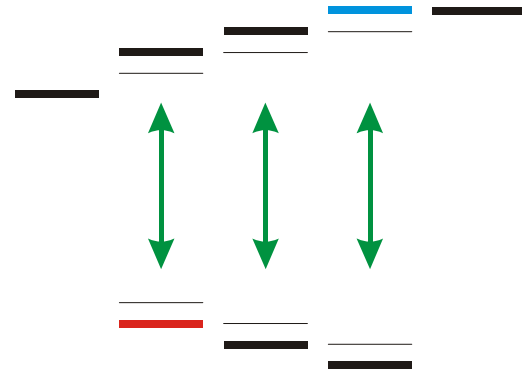
$$|F = 1, m_F = -1\rangle$$

- Identical Zeeman shift
- Identical Stark shift
- Identical light shift (for large detuning)

Radio Frequency (RF) and Micro Wave (MW) fields can couple differently

On chip: local RF and MW field for manipulation

Linear polarized micro wave



AC-Zeeman shift:

$$\Delta E = \pm \frac{\hbar \Omega_R^2}{4\Delta}, \quad \text{with } (|\Delta| \gg \Omega_R)$$

$$\hbar \Omega_R \sim \mu_B \cdot B_{MW}$$